Instructor's Guide

Instructor Resources
Worksheets

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Arizona State University
Diné College
Kennesaw State University

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Module Organization

Kéyah Math modules are organized by topic, location and geographic theme. Each module integrates geoscience and mathematical concepts within applications.

The specific modules and their mathematical levels are given in the tables below. You can click the link to go to the module resources on the website, detailed in this guide. Table 1 provides an organization of the modules themselves, while Table 2 shows the requisite mathematics per level. The module section gives detailed information about the use of the materials.

Organization of Kéyah Math Modules

Topic	Level	Module Name
Demonstration of Kéyah Format	Level 0	Age of the Universe
Stream Flow	Level 1	Stream Flow for the Animas River
Stream Flow	Level 2	Snow Melt & Stream Flow for the Animas River
Earthquakes	Level 2	Location of the Epicenter of an Earthquake
Volcanic Processes	Level 2	Sunset Crater
Age of the Earth	Level 2+	Age of the Earth
Age of the Earth	Level 4	Geochronology in the San Juan Mountains
Impact Processes	Level 2	Meteor Crater
Impact Processes	Level 4	Age of the Earth
How Big is the Earth?	Level 2	The Size of the Earth, Estimated in Arizona
How Big is the Earth?	Level 3	Mass & Density of the Earth and Size, Mass, & Density of the Earth
Layers of the Earth	Level 3	Layers of the Earth

Table 1

Requisite Mathematics per Level

Level	Mathematical Topic
Level 1	Pre-Algebra, Substitution into Formulas, Computation, Simple Geometry
Level 2	Algebra with Equations (not Functions), Solving Equations, Reading Graphs, Geometry
Level 3	Algebra with Functions, Evaluating Algebraic Functions, Solving Equations, Graphing
Level 4	Pre-Calculus, Algebraic & Exponential Functions, Evaluation, Graphing, Geometry

Table 2

Website Organization

Click the link to go to website page.

Module Map **Map with Modules on Location**

Topic Map **Map of Topics, by Geoscience Reference**

Topic Menu **List of Modules by Level**

II. Kéyah Math Project

A. Background

Project Description:

The ***Kéyah Math Project*** has developed a series of versatile online activities in mathematical geoscience, using the natural and cultural landscapes of the Southwest United States as context and setting. These place-based exercises are available to enhance any undergraduate geoscience course, and may be of particular interest to students and teachers with cultural ties to the Southwest, including American Indian and Hispanic students and teachers.

Kéyah is the Diné (Navajo) word for homeland-- it literally means that which is connected to one beneath one's feet. We use this name to emphasize the connection to Southwest places.

Fourteen activities address five levels of mathematical content and are partitioned among seven topics. (Note that one of these--Age of the Universe-- is a demonstration of the Kéyah Math format and is identified as Level Zero.) Kéyah Math activities are typically not found in introductory geoscience textbooks, nor are they readily available commercially in ancillary materials. They draw on data-rich examples from the geological and cultural landscapes of the Southwestern United States, including the lands of several American Indian nations. Indigenous cultural and scientific experts have participated directly in the development of these activities. The versatility and accessibility of these activities will enable any number of them to be integrated into any basic Earth science or geology course at any level, regardless of the textbook or laboratory manual used.

Modules are self-contained lessons, with various mathematics levels, that draw on both scientific principles of Geoscience and concepts or skills of mathematics. Students have access to a Mathematics Tool set, to be used in solving various data-rich problems, or may use a calculator if a computer is not available. Instructors may use the tools to demonstrate concepts or assist students in problem-solving. Each module leads students to conclusions about the Geoscience topic and use of Mathematics in solving problems.

This guide contains instructions and answers for instructors, and worksheets to use with students to record answers or work independently from a computer, if one is not available. An inquiry-based approach, instructors should spend time in discussion with students using the Warm-up Questions and Questions for Follow-up. Appropriate for small group instruction or individual student work, the modules additionally allow instructors to assign different level work to students within the same classroom.

Instructor sheets and student worksheets are available in a format that will allow instructors to modify and add to the basic module to fit their Standards and curriculum. Links to additional information and related activities are provided to enrich the students' exploration of the topics.

Students in the southwest can benefit from instruction that includes local data. Using the modules as a base, instructors can easily adapt the modules to include local data obtained from the Geoscience around the students; streams, rocks, and formations.

The Kéyah Math team plans to continue developing place-based quantitative geoscience activities, and welcomes your comments and recommendations (Contact Team).

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B. Pedagogical Concerns:

Recommendations of the Greater Geoscience-Education Community:
In 1996, the National Science Foundation published the "George Report," a comprehensive review of undergraduate education in all areas of science, technology, engineering, and mathematics (George et al., 1996) that in turn inspired communities of educators in the various disciplines to conduct their own reviews. Later that same year, in response to the "George Report," the Shaping the Future conference on innovation in undergraduate Earth science education (Ireton et al., 1997), also sponsored by NSF, was convened by the American Geophysical Union and the Keck Geology Consortium.

A panel at this conference addressed the question of increasing diversity, recruitment, and retention of students in the Earth and space sciences. The foremost recommendation of this Diversity Panel (which included the PI of this project), to college and university instructors, echoes the sentiments of Native educators discussed above:

"Place Earth system science principles and problem-solving methods in the context of the local environment, which helps students connect the relevance of Earth system science to their lives…." (Ridgway et al., 1997).

The place-based, culturally-responsive framework of the Kéyah Math modules is fully concordant with this recommendation.

The recent report, Blueprint for Change (Barstow et al., 2002), the proceedings of a major recent NSF-funded workshop in geoscience education reform involving a diverse group of educators (including the PI of this project), also lists a number of recommendations for improving curricula and instructional materials. These, though formally addressed to the K-12 community, are equally relevant at the lower-division undergraduate level. The proposed Kéyah Math project responds directly to several of these recommendations, specifically that new curricula and instructional materials should:

- Engage students by means of "dynamic learning opportunities that are relevant to students' lives and communities;
- "Be inquiry-based;
- "Illustrate how Earth and space science reflects the contributions of and is relevant to diverse populations;
- "Provide a forum for the development of skills in math…." (Barstow et al., 2002)

Benefits of Using Kéyah Math:

Results of the project evaluation show:

1. Kéyah Math bolsters the interest and capabilities of all students in the geosciences through the use of scientific inquiry and current scientific data.
2. Kéyah Math attracts the interest of Native American students in particular, through the use of data and case studies taken from familiar, culturally-significant localities and contemporary issues of significance to their communities.
3. Kéyah Math improves the quantitative skills of Native American and other minority science students at an early stage in their undergraduate programs, better preparing them for professional careers in the geosciences.
4. Kéyah Math enhances the global infrastructure for geoscience education through universal web-based dissemination.

Full evaluation report available (see Kéyah Math website)

III. Kéyah Modules

This section includes title, level and a brief description of the modules. Click on the Instructor Sheets link to go to that module section for a complete set of instructions for using the particular module.

A. Demonstration of Kéyah Format

Instructor Sheets

0 Level 0 Age of the Universe KM#0

Brief Description: This study provides an introduction to the format for the Kéyah Math studies by example. The age of the Universe is computed illustrating the structure of a module and use of the applets.

Mathematical Content: Basic algebra, linear equations, graphs ***Kéyah Math Student Worksheets***

The link above right will provide the module questions in a format for students without computer for Math Tools.

Objectives: To determine:

1. a linear model for distance of a galaxy from the earth against the velocity of recession;
2. an estimate for the age of the universe.

Mathematical Concepts/Skills

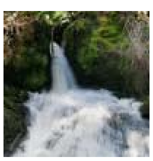
Graphing Data Points
Scientific Notation
Linear Equations
Unit Conversion
Solving Simultaneous Equations

LEVEL 1 MODULE:

Pre-algebra; arithmetic; substitution into formulas; computation; simple geometry

B. Stream Flow

Instructor Sheets



1 Level 1 Stream Flow for the Animas River KM#1

Brief Description: This study explores a very simplified method for predicting streamflow, or the amount of water that flows through a stream. After an introduction, the area of its watershed is computed. Then, given the average amount of precipitation, the volume of water contributed to the watershed is determined. This information is used to find the amount of water that goes into the stream, or the annual streamflow.

Mathematical Content: Area, Volume, Unit Conversion ***Kéyah Math Student Worksheets***

The link above will provide the module questions in a format to use without computer for Math Tools

Objectives: To determine:

1. The area of the drainage basin
2. The average monthly streamflow of the Animas River

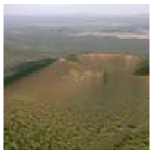
Mathematical Concepts/Skills

Unit Conversion
Calculate area and volume
Percent of Total
Linear and Time Conversion

LEVEL 2 MODULES:

Algebra with equations (not functions); solving equations; reading graphs; geometry

C. Volcanic Processes   Instructor Sheets



2 Level 2   Volcanic Ejecta from Sunset Crater   KM#2

Brief Description: This module is a study of ejecta from Sunset Crater, just north of Flagstaff Arizona. The velocity of bombs ejected from the crater during its eruption is determined using equations given to students.

Mathematical Content: Basic algebra; solving equations   ***Kéyah Math Student Worksheets***

The link above will provide the module questions in a format to use without computer for Math Tools

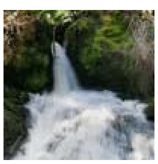
Objectives: To determine:

1. The velocity of bombs ejected from the crater during eruption;
2. Use given equations to determine velocity.

Mathematical Concepts/Skills

Order of Operations
Notation
Using Formulas
Solving for an Unknown

D. Stream Flow   Instructor Sheets



2 Level 2   Snow Melt & Stream Flow in the Animas River   KM#3

Brief Description: This study explores a simple way to predict streamflow, or the amount of water that flows through a stream. After an introduction, there are 3 parts, predicting temperature, precipitation (rain and snow), then the volume of water over the watershed is computed and used to predict streamflow. Then, given the average amount of precipitation, the volume of water contributed to the watershed is determined. This information is used to find the amount of water that goes into the stream, or the annual streamflow.

Mathematical Content: Area, Volume, Unit Conversion, linear equations.   ***Kéyah Math Student Worksheets***

The link above will provide the module questions in a format to use without computer for Math Tools

Objective: To determine:

1. How snow melt affects streamflow in a certain time period.

Mathematical Concepts/Skills

Interpreting Tables
Computation
Scientific Notation
Finding Volume, Total Volume

E. How Big is the Earth?

Instructor Sheets

2 Level 2 Measuring the Size of the Earth KM#4

Brief Description: The goal of this study is to learn how Eratosthenes estimated the circumference of the earth. After explaining the geometry used by Eratosthenes to estimate the circumference of the Earth, students use this information to compute the radius and volume.
Mathematical Content: Geometry (radius and circumference of a circle; volume of a sphere); interior angles.

Kéyah Math Student Worksheets

The link above will provide the module questions in a format to use without computer for Math Tools

Objectives: To determine

1. The circumference of the earth mathematically using Eratosthenes' method.

Mathematical Concepts/Skills

Basic Geometry (Angles, Circumference)
Estimation
Computation
Using Formulas
Volume of a Sphere
Scientific Notation

F. How Big is the Earth?

Instructor Sheets

2 Level 2 Measuring the Size of the Earth from Arizona KM#5

Brief Description: This study uses the method of Eratosthenes with local information from Arizona to estimate the circumference of the earth. This estimate is then used to compute the Earth's radius and volume.
Mathematical Content: Geometry (radius and circumference of a circle; volume of a sphere); interior angles

Objectives: To determine

2. The circumference of the earth mathematically using Eratosthenes' method for two locations in Arizona.

Mathematical Concepts/Skills

Basic Geometry
Estimation
Computation
Using Formulas
Volume of a Sphere
Scientific Notation
Solve for Value

G. Earthquakes

Instructor Sheets

2 Level 2 The Epicenter of an Southwestern Earthquake KM#6

Brief Description: This study examines the speed of P and S waves emitted from an earthquake, Equations relating distance traveled and speed are written, then the equations are used to find the epicenter as the intersection of three circles, each centered at a seismic station.
Mathematical Content: Basic algebra; equation of a line; radius of a circle.

Objectives: To determine:

1. The equation for how far each type of wave traveled in t seconds;
2. The difference in arrival time for the sound waves
3. Find the epicenter of an earthquake

Mathematical Concepts/Skills

Notation	Solve for Value
Distance	Calculation
Solve Equation for Variable	Circles – diagram center & radius

H. Impact Processes

Instructor Sheets

2 Level 2 Meteor Crater KM#7

Brief Description: This study examines Meteor Crater, Arizona, and the size of the meteorite that formed it. After an introduction to impact processes, students are guided through a sequence of steps that involve relatively simple formulas from physics to find the size of the meteorite. Specifically: a formula relating kinetic energy and diameter of the crater is used to find kinetic energy released on impact; a formula relating KE and mass is used to find the mass of the meteorite; a formula relating mass density and volume is used to find the volume of the meteorite; and finally, a formula relating spherical volume to radius is used to find the diameter of the meteorite.
Mathematical Content: Basic algebra, solving simple equations

Objectives: To determine:

1. The kinetic energy released and mass of the meteorite on impact that formed Meteor Crater;
2. The volume and diameter of the meteorite.

Mathematical Concepts/Skills

Using a Formula	Solve for an Unknown
Substitution	

I. Age of the Earth

Instructor Sheets

2 Level 2+ Ages of Rocks and the Earth KM#8

Brief Description: This study explores methods for dating rocks using radioactive decay to find the age of rock from the San Juan Mountains and the age of the Earth. A basic decay equation for the Rubidium-Strontium isotope system is given and applied to date rock samples from Electra Lake, Southwest Colorado. Then this equation is then used to date a meteorite containing rubidium and strontium to estimate the age of the Earth.
Mathematical Content: Basic algebra; exponential equations; logarithms

Objectives: To determine:

1. The age of rock using radioactive decay;
2. The age of the earth using a meteorite.

Mathematical Concepts/Skills

Plot Points	Find Value from Equation
Substitution	Slope of a Line
Find Linear Equation	Find Line Graphically

Linear Regression

The ***Kéyah Math***

LEVEL 3 MODULES:

Algebra with functions; evaluating algebraic functions; solving equations; graphing

J. How Big is the Earth? Instructor Sheets

3 Level 3 Mass & Density of the Earth KM#9

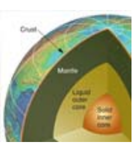
Brief Description: This study is a follow-up to the size of the Earth modules. It uses estimates of the radius and volume of the Earth to compute its mass and density. Methods of Newton using the Law of Gravitational Attraction and density equations are used.
Mathematical Content: Geometry (radius and circumference of a circle; volume of a sphere); interior angles; Newton's Law of gravitational attraction; formulas for density and force; solving equations involving kinetic energy, mass and volume

Objectives: To determine:

1. An estimation for the radius and volume of the Earth;
2. To use estimates to compute the mass and density of the Earth using methods of Newton.

Mathematical Concepts/Skills
Scientific Notation
Law of Gravitational Attraction
Using Formulas to Solve for Unknown
Volume of a Sphere

K. Layers of the Earth Instructor Sheets



3 Level 3 Layers of the Earth KM#10

Brief Description: This study explores using graphs of travel times of seismic waves to discover a layering structure of the Earth. After an introduction, and using results for the diameter of the earth found in either module on the size of the Earth, the student is given information about the travel time of two seismic waves to a station a known distance from the epicenter of an earthquake. The student computes the velocity of the first seismic wave and the distance the second wave travels. From this information it is now possible to deduce the path of the second wave and conclude that the earth is layered. The depth of this layer and the radius of the inner barrier are also computed.
Mathematical Content: Basic algebra (distance, rate and time); geometry; Snell's law

Objectives: To determine:

1. The velocity of a seismic wave and distance traveled to pinpoint the layers of the earth;
2. The depth of a layer and radius of the inner barrier.

Mathematical Concepts/Skills
Scientific Notation
Calculating Velocity
Geometry of Angles

L. How Big is the Earth?

Instructor Sheets

3 Level 3 Size, Mass & Density of the Earth KM#11

Brief Description: The goal of this study is to learn how Eratosthenes estimated the circumference of the earth. After explaining the geometry used by Eratosthenes to estimate the circumference of the Earth, students use this information to compute the radius and volume. The students then apply the same method to local Arizona data and derive another estimate for the circumference and volume of the Earth. Finally, this estimate is then used to compute the Earth's density.
Mathematical Content: Geometry (radius and circumference of a circle; volume of a sphere); interior angles; Newton's Law of gravitational attraction; formulas for density and force; solving equations

Objectives: To determine:

1. An estimation of the circumference of the Earth using methods of Eratosthenes;
2. An estimate of the Earth's density.

Mathematical Concepts/Skills

Angle Geometry
Circumference
Using a Formula
Volume of a Sphere
Law of Gravitational Attraction
Mass and Density
Force
Solving Equations

LEVEL 4 MODULES:

Pre-calculus; algebraic and exponential functions; evaluation; graphing; geometry

M. Age of the Earth

Instructor Sheets

4 Level 4 Geochronology in the San Juan Mountains KM#12

Brief Description: This study explores methods for dating rocks using radioactive decay to find the age of rock from the San Juan Mountains and the age of the Earth. A basic decay equation for the Rubidium-Strontium isotope system, is derived and applied to date rock samples from Electra Lake, Southwest Colorado. Then this equation is used to date a meteorite containing rubidium and strontium to estimate the age of the Earth.
Mathematical Content: Basic algebra; exponential equations; logarithms; linear regression.

Objectives: To determine:

1. Decay constants and isochron diagram to find the age of rock samples;
2. The date of a meteorite used to estimate the age of the Earth.

Mathematical Concepts/Skills

Plot Sample Data Points
Linear Regression
Slope of a Line
Solve for Unknown
Exponential Equation

N. Impact Processes

Instructor Sheets

4 Level 4 Impact Processes at Meteor Crater (Advanced) KM#13

Brief Description: This study examines Meteor Crater, Arizona, and the size of the meteorite that formed it. After an extensive introduction to impact processes, students are guided through a sequence of steps that involve formulas from physics to find the size of the meteorite. Specifically: a formula relating kinetic energy and diameter of the crater is found using power regression on real data. This is used to find kinetic energy released on impact; a formula relating KE and mass is used to find the mass of the meteorite; a formula relating mass density and volume is used to find the volume of the meteorite; and finally, a formula relating spherical volume to radius is used to find the diameter of the meteorite.

Mathematical Content: Algebra, power and linear equations, linear regression, solving equations involving kinetic energy, mass and volume

Objectives: To determine:

1. The size of the meteorite from kinetic energy and diameter of Meteor Crater;
2. The volume of the meteorite that impacted the Earth to form Meteor Crater.

Mathematical Concepts/Skills

Using Formulas
Deriving an Equation
Modeling Density
Regression Models
Solving Equations

IV. *Demonstration of Kéyah Format* Instructor Sheets

0 Level 0 Age of the Universe KM#0

Brief Description: This study provides an introduction to the format for the Kéyah Math studies by example. The age of the Universe is computed illustrating the structure of a module and use of the applets.

Mathematical Content: Basic algebra, linear equations, graphs Kéyah Math Student Worksheets

The link above right will provide the module questions in a format for students without computer for Math Tools.

Objectives: To determine:
1. a linear model for distance of a galaxy from the earth against the velocity of recession;
2. an estimate for the age of the universe.

Keyah Math Instructor Directions

This first demonstration module follows step-by-step directions for learning how to use the modules. You can assign this as a preliminary module online or use the worksheets to walk through the material with your students.

Students should be encouraged to use an inquiry approach and a journal to record their observations and answers, whether using a computer or paper journal. The worksheet link below will provide worksheets for students without computers.

Be sure to remind students to write in complete sentences and express ideas so that others can understand. Save work and keep a window open throughout this study if using a computer so you can easily record other questions and answers.

Use questions for thought as a preliminary discussion about the topic.

Questions for thought:

1. What do you think of when you think of the universe?
2. What do you know about the age of the universe?
3. How do you think scientists might obtain an estimate of the age of the universe?

Students should write a brief response to these questions in their journal.

The information that follows is used in the exploration of the age of the universe. A list of questions at the end of the section will help this exploration.

Notation:

Basic units

- Because astronomical distances are so large, astronomers like to use a large measure, a parsec. One parsec (pc) is approximately 3.26 light years, or 3.1×10^{13} kilometers (km). (One light year is the distance that light travels in one year.)
- In this study, the distance d of a galaxy from Earth is given in Megaparsecs (Mpc)
- Mega means million, so 1 Mpc = 1×10^{6} pc.
- 1 Mpc = 3.1×10^{19} km

Assumption

The relation between distance and velocity is linear.

$d(x10^3)$	$v(x10^3)$
0.028	2.7
0.076	4.2
0.108	10.5
0.136	14.1
0.153	10.5
0.226	13.2
0.283	19.8
0.359	28.2
0.363	20.7
0.408	29.4
0.438	31.8
0.472	44.4
0.476	32.1
0.476	37.2
0.493	33
0.556	34.5
0.639	46.5

Constructing the Hubble Diagram
The data points to the left are taken from a plot of redshift z of receding galaxies against their distance d from Earth (Galaxies and Cosmology, Jones and Lambourne, page 243) with the second row indicating

redshift x velocity of light = velocity of recession of galaxy = v

The numbers in the distance column give distance d of the galaxy from Earth in Megaparsecs x 10^3 (Mpc); the velocity is kilometer per sec x 10^3. So, for example, a galaxy which is 0.028 x 10^3 Mpc from Earth is receding at a velocity of 2.7 x 10^3 km/sec.

Linear Models
In this part, you will find a linear function to express the relationship between distance and velocity. Specifically you will:

* express velocity as a function of distance from the earth,
* find the Hubble constant, and
* determine how fast the Whirlpool Galaxy is receding from us.

For this demonstration module, ***instructions are provided in italics and solutions are in blue italics.***

Problem Set *(Round off to three decimal places for this work.)*

The applets needed to complete this problem are provided in the text for this Demonstration module. Applets are available on the Kéyah Math website in the Math Tool Chest. Usually, you will need to follow these steps. (You may choose to use graphing calculators to do this work.)

* *Scroll down to "Tool Chest" on the menu to the left of the screen.*
* *Click "linear regression" (since we are looking for a linear function).*

The linear regression applet will open in a separate screen (see next page for view).

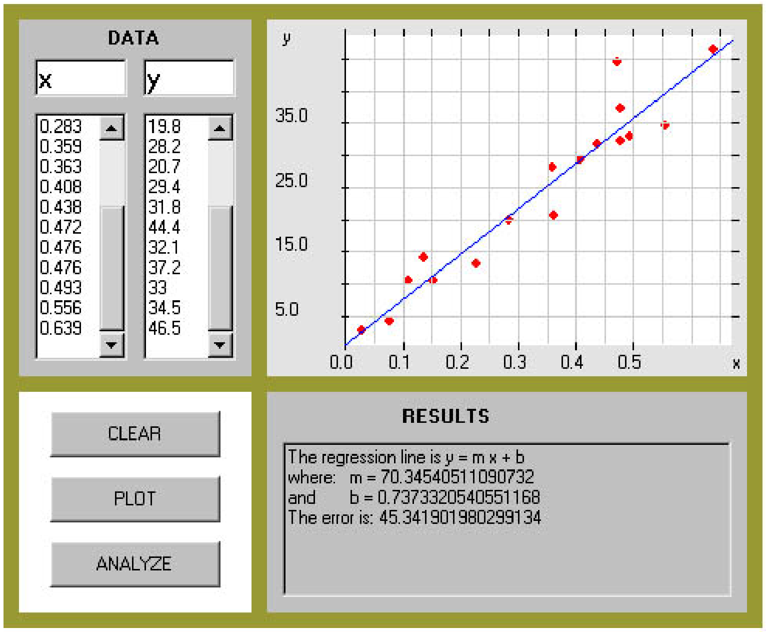
Question 1: Plot the points corresponding to the data in Table 1. The first coordinate is distance from the earth; denote this by d (units are 10^3 Mpc). The second coordinate is velocity of recession of galaxy, units 10^3 km/sec.

You can copy each column from the table and paste it into the appropriate column of the applet. If using a graphing calculator, enter the data list for the coordinates. Remember to set the screen view and significant digits.

Question 2: Now find the line of best fit. (The resulting graph is called a **Hubble Diagram**.)

The equation of the line is shown in lower right window of the applet screen:
$y = 70.345x + 0.737$.

This is the line calculated to fit closest to the date points provided; you can ask students working on paper to sketch where they think this line would go, then plot using the calculator or computer. Discuss their choice of location and how close they estimated the resulting line.



Enter the data provided in Table 1 into the columns on the applet screen; enter the d value in the "x" column, then enter the v value in the "y" column. You can copy each column from the table and paste it into the appropriate column of the applet.

Click "Plot", you should see the points on the screen at left, without results.

Click "analyze" on the applet containing the plotted data and see the screen as shown, with the slope and intercept as well as the error.

Question 3: What is the slope of this line? What are the units?

The slope is indicated by the constant m, it represents the change in y divided by the change in x. The units for the numerator are the units for velocity, 10^3 km/sec; the units for the denominator are units for distance, in this case 10^3 Mpc; thus the units for the slope are (km/sec)/Mpc.

m = 70.345 (km/sec)/Mpc.

Your answer is the Hubble constant, denoted by H_0. Here's what it means: if a galaxy is 10 Mpc away from us, it is receding at the rate of 10 x 70.345 km/sec = 703.45 km/sec; if a galaxy is 15 Mpc away, it is receding at the rate of 15 x 70.345 km/sec = $10^5$5.175km/sec; etc.

Question 4: The Whirlpool Galaxy is approximately 30 x 10^6 light years away from us. (One light year is the distance that light travels in one year.) How fast is it receding from us?

We need to use the same units for all measurements, so converting 30 x 106 light years to Mpc, we get 9.2 Mpc.

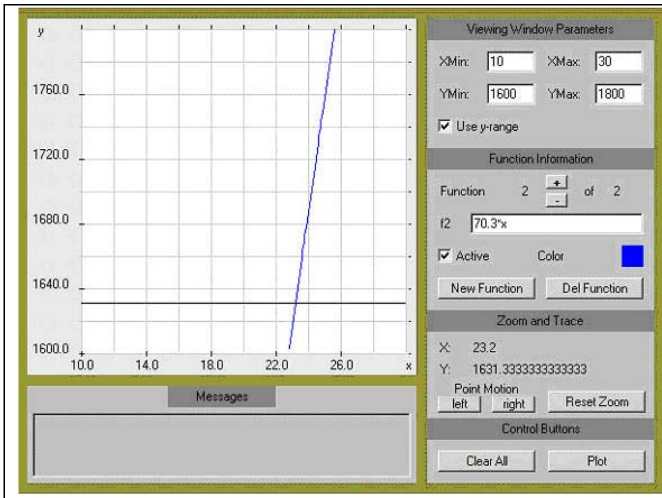
Now use the Hubble constant to answer the question.

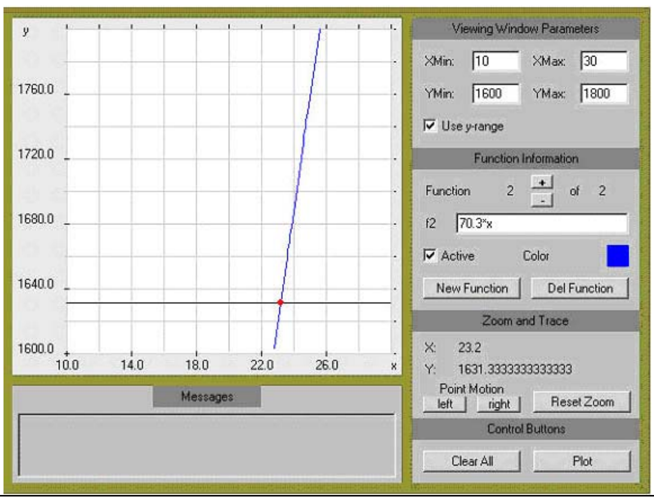
The galaxy is receding at the rate of 9.2 x 70.345 = 647.174 km/sec.
(This calculation can be done using MathPad or your calculator).

Information:
We can use the Hubble constant to approximate the age of the universe. The modified equation from #1 (ignoring the very small constant, assuming that when d = 0, then v = 0, and rounding the Hubble Constant to one decimal place),
v = 70.3d, known as **Hubble's Law**.

Question 5: A Black Hole, Perseus A, has been detected traveling away from us at the rate of 1631 km/s. How far away is it?

This could actually be solved easily by dividing 1631 by 70.3, however we will use the Plot-Solve Applet to show how it can solve equations. The equation we need to solve is 1631 = 70.3d, for the distance d.





Click "Tool Chest" on the left menu, and then click "Plot-Solve." You'll see the screen to the left.

Now click "New Function" and type in the left side of the equation, 1631; then click "Plot."

*Next, click "New Function" and type in the right side of the equation (remember to use * for multiplication, and use x for d), 70.3*x; then click "Plot again".*

Change the y-range. Enter 1600 for "YMin" and 1800 for "YMax"; this range includes 1631, which is what we want to see.

We also want to see the solution, so you'll need to change the x-range also: let's guess and enter 10 for "XMin" and 30 for "XMax."

Now, click "Plot" again. You should see the screen shown at left. The solution to our equation is the x-coordinate of the intersection of the two graphs.

Place the point of the arrow on your cursor right on the intersection of the two graphs and click "Plot." The x-coordinate can be read in the window to the right of the screen, you should see

x = 23.2.

So, the black hole is 23.2 Mpc away from Earth.

Question 6: Recall the well-known formula for distance, **distance = (rate of speed) x (time traveled)**, or **d = vt,** and **substitute this into the Hubble Law for d.** Simplify the result and **solve for time, t.**

v = 70.3d — *Cancel the v from both sides to get 1 = 70.3t*
v = 70.3(vt) — *Solve for t to get t = 1/70.3*

In order to keep the same units and express the age of the universe in years, the Hubble constant, H_0 = 70.3 km/s (per Mpc), expressed in Mpc/yr (per Mpc) is 72.5 x 10^{-12} Mpc/yr (per Mpc).

Question 6: Using this value for the Hubble constant (replace 70.3) and the answer to Question 6, compute t. This answer is a very good approximation to the current age of the universe.

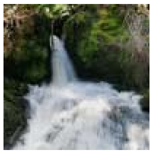
t = 1/(72.5 x 10^{-12}) = 13.8 x 10^9 years, or 13.8 billion years
(Note the units here: the Mpc's cancel leaving only years.)

See: Kéyah Math Worksheets for Students

V. <u>LEVEL 1 MODULE</u>:

Pre-algebra; arithmetic; substitution into formulas; computation; simple geometry

A. Stream Flow — Instructor Sheets



1 Level 1 Stream Flow for the Animas River KM#1

Brief Description: This study explores a very simplified method for predicting streamflow, or the amount of water that flows through a stream. After an introduction, the area of its watershed is computed. Then, given the average amount of precipitation, the volume of water contributed to the watershed is determined. This information is used to find the amount of water that goes into the stream, or the annual streamflow.
Mathematical Content: Area, Volume, Unit Conversion Kéyah Math Student Worksheets
The link above will provide the module questions in a format to use without computer for Math Tools

Objectives:

- Unit Conversion
- Calculate area and volume

Introduction

This is a study about finding the average amount of water that flows through the Animas River each month. The river has its headwaters above Silverton, Colorado, and flows south to Farmington, New Mexico, where it empties into the San Juan River. This study is concerned with the Upper Animas from its headwaters to Durango. It illustrates (in a simplified way) how scientists can use historic data to predict how much water runs through a stream in a certain time period. This is called *streamflow*, it can be recorded using various units, for example, in cubic feet per second (cfs), or cubic meters per second or cubic feet per month, etc. Basically, it is measured by volume per time increment.

If you measure streamflow at some point on a stream as 500 cfs, this means that 500 cubic feet of water passes through that point every second. For example, the streamflow for the Mississippi River at Baton Rouge is 211,000 cfs whereas the streamflow for the Colorado River below the Laguna Dam (on the Arizona/ California border) is 398 cfs. Geologists are interested in streamflow because it affects amounts of sediment carried by streams (this is somewhat dependent on streamflow) and how this might change stream beds and land formation.

Warm-up Questions

Use questions for thought as a preliminary discussion about the topic.

- How much water do you think flows down a stream near you every day?
- How could you measure actual streamflow for this stream?
- Why is it important to predict streamflow for your river or stream?

Using Math to Find the Streamflow for Animas River

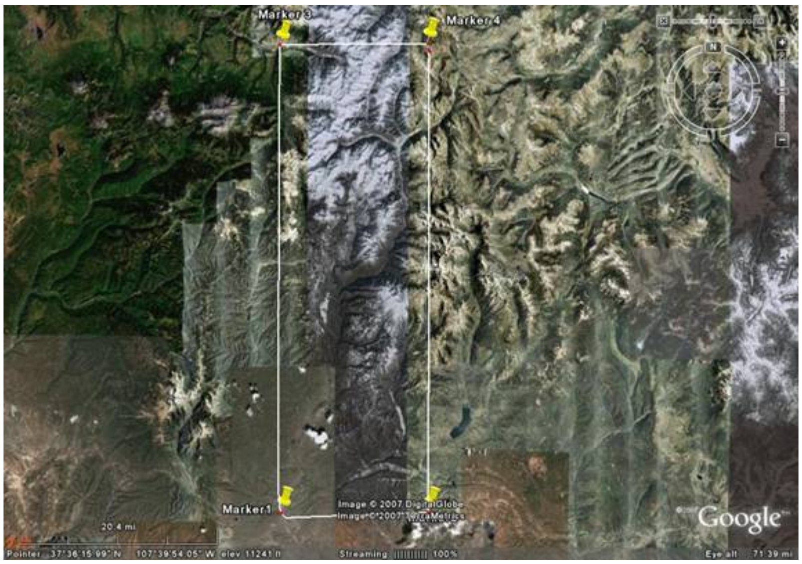
The questions that follow will lead you to help students figure the average monthly streamflow for the Animas River at Durango. The first step is to estimate the area of the drainage basin.

There are two versions of this module: the first version provides a map of the drainage basin and its approximate area; the second version uses Google Earth and asks you to calculate your own estimate of the area. Either version is available on the website.

Using Math to Find the Streamflow for Animas River (Version 1)

The questions that follow will lead your students to figure the average monthly stream flow for the Animas River at Durango. We measure this in cubic feet per month, then convert to cubic feet per second, the most common units used for stream flow in this country.

The first step is to estimate the area of the drainage basin. Figure 1 shows the approximate drainage basin for the Animas from source to Durango. Of course, the actual watershed is not rectangular but this shows the approximate region.



Information you'll need to answer this question is bulleted below. Refer to the figure shown, and then answer the question below the figure.

- The area of the drainage basin, or watershed, for the Animas from source to Durango is roughly 700 square miles.

Source (Note: This was obtained by computing the area of the rectangle shown. The USGS web site Water Watch (http://water.usgs.gov/waterwatch/?m=real&r=co) shows the actual watershed is 692 square miles. You can check this out by going to the site and locating the dot for Durango in the southwest portion of the state. More information on the watershed is given in the source listed below):
http://www.nmenv.state.nm.us/swqb/Projects/SanJuan/TMDL1/03.pdf

- The total average annual precipitation, including water from snow, over this region is 22.17 inches.

Source (Note: The figure above was obtained by averaging data from these two sources: Monthly weather averages for Silverton, CO from Weather.com and Monthly Climate Summary from the Western Regional Climate Center

Using math to estimate average stream flow for the Animas River

Question 1: What is the area of the watershed in square feet?
(1 mile = 5,280 feet, so 1 square mile = $(5280 \text{ ft})^2 = 5280^2$ square feet)

Answer: 1.95×10^{10} square feet

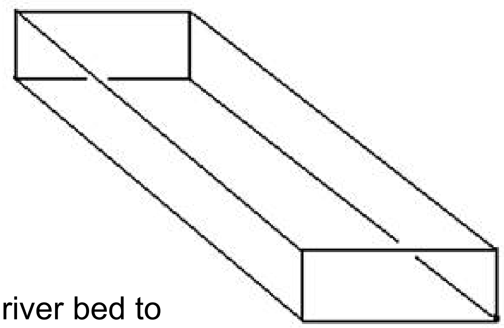
Question 2: Convert the annual amount of precipitation from inches to feet.
(1 foot = 12 inches)

Answer: 1.85 feet

Question 3: Now, find the total volume (in cubic feet—use your answers to #1 and 2) of water from rain and snow that falls on the watershed each year.
Think of the drainage basin as a giant bucket with a rectangular base with area 700 square miles and height the amount of precipitation over the area, 22.17 inches. (See figure below.)
(Volume = Area of Base x Height)

Answer: 3.61×10^{10} ft^3

The base of the figure is the rectangular watershed, and the height is the depth of water from precipitation.



Question 4: Only 74% of the rain actually reaches the river bed to contribute to its stream flow (all the rest of the water is evaporated or diverted for other uses). **What is the annual stream flow for the Animas River?**

Answer: 2.67×10^{10} ft^3

Question 5: On the average, how much water flows down the river each month? Each second?

(1 year = 365.2 days, 1 day = 24 hours, 1 hour = 60 minutes, and 1 minute = 60 seconds)

Answer: 0.22×10^{10} ft^3/month; 846.19 ft^3/sec

Your answer to # 5 is the average stream flow for the Animas River at Durango; the units should be cubic feet per second, or cfs.

Looking back at answers

These questions provide you with an extension to the module, particularly for levels teaching.

- Do you think that the methods used here would be accurate for predicting future stream flow?
- Do you think that you can accurately predict daily, or monthly, stream flow from annual stream flow, particularly for the Animas River? Why?
- What variations in precipitation might affect monthly stream flow?
- How would variations in stream flow affect the stream bed, or the land around the stream?

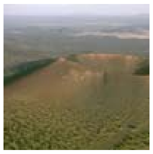
For an interesting extension, go to:
http://pubs.usgs.gov/of/1992/ofr92-129/hcdn92/hcdn/ascii/monthlya/region14/09361500.amm
find the average of the data given there, and compare answers.

Since much of the Animas River watershed lies in the San Juan Mountains at elevations from 8,000 to 14,000 feet, snow and snow melt drastically affects its stream flow. Again, see the website listed above. For a better look at how Animas stream flow is affected by snowfall, go to KM Study #2, "Snowfall and the Animas River Streamflow."

VI. LEVEL 2 MODULES:

Algebra with equations (not functions); solving equations; reading graphs; geometry

B. Volcanic Processes — Instructor Sheets



2 Level 2 Volcanic Ejecta from Sunset Crater KM#2

Brief Description: This module is a study of ejecta from Sunset Crater, just north of Flagstaff Arizona. The velocity of bombs ejected from the crater during its eruption is determined using equations given to students.

Mathematical Content: Basic algebra; solving equations   Kéyah Math Student Worksheets

The link above will provide the module questions in a format to use without computer for Math Tools

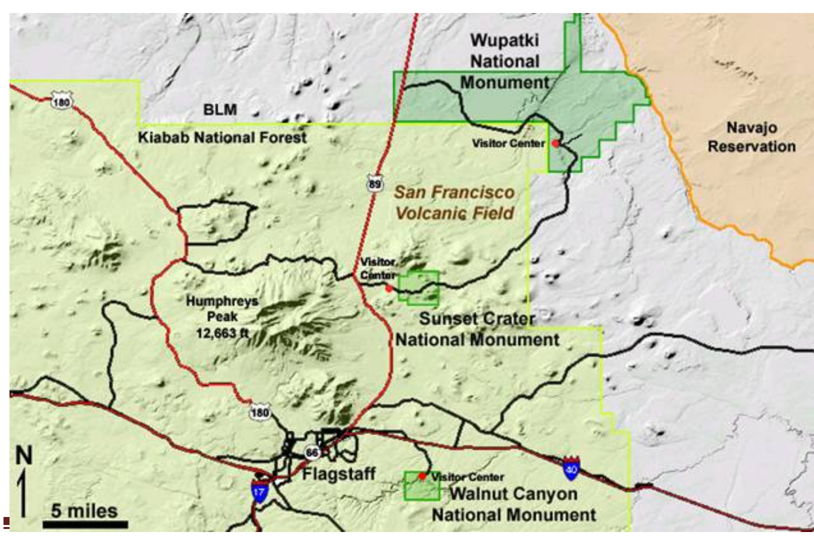
Objectives:

- The velocity of bombs ejected from the crater during its eruption is determined using given equations.

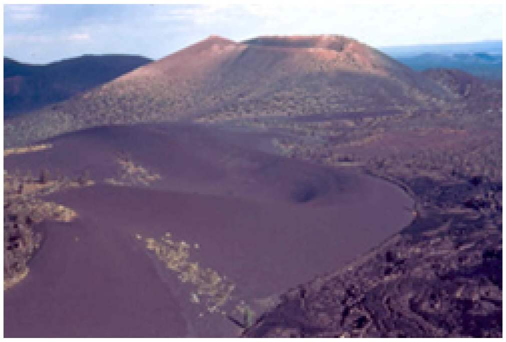
Introduction

Sunset Crater is located about 15 miles (25 kilometers) northeast of Flagstaff, Arizona (see map below), along the east edge of the San Francisco volcanic field. Volcanic activity in this area was initiated about 6 million years ago and ended about 1000 years ago with the eruption of Sunset Crater (Reynolds et al., 1986; Holm and Moore, 1987; Duffield, 1997. Early eruptions were dominated by the construction of a composite volcano and related domes and flows while later eruptive phases were mostly eruptions of basaltic magmas that formed cinder cones and associated lava flows and spatter cones. More than 600 of these cinder cones have been documented in this region (Duffield, 1997) with Sunset and SP Craters being several of the more well known cones.

Initial eruptions at Sunset Crater ejected volcanic material ranging from fine ash to the size of watermelons (bombs); materials ejected during an eruption are referred to as **tephra**. Erupted bombs associated with this event were formed when sticky blobs of magma were ejected from the vent by gas-charged eruptions. As the hot and sticky masses spun in the air they took on different shapes and forms, some resembling footballs. These eruptions produced a mantle of loose, black material that accumulated around the central vent constructing a nearly symmetrical cone about 1,000 feet (300 meters) high and 1 mile (1.6 kilometers) in base diameter with a 400-foot-deep crater. All of the erupted material associated with this event may have blanketed more than 800 square miles, but presently about 122 square miles are covered with erupted tephra (Holm and Moore, 1987).



As part of the formation of the Sunset Crater cinder cone, basaltic lava flows (Bonito lava flow and Kana-a flow) erupted along cracks and fissures at the base of Sunset Crater. The Bonito lava flow traveled to the north-northwest for around 3,700 feet (1,130 meters) whereas the Kana-a flow migrated from the east of the cone northeast for about 4 miles (6.4 kilometers). Holm and Moore (1987) also describe small cones to the southeast of the main cinder cone.



A shaded relief computer image of Sunset Crater and other features in the area image from the United States Geological Survey;

Sunset Crater as viewed to the south. Note the lava flows and other cinder cones in the field (http://www.nps.gov/sucr/naturescience/volcanoes.htm).

Warm-up Questions

Use questions for thought as a preliminary discussion about the topic.

1. How fast do you think a bomb ejected from the Sunset eruption would travel?
2. How far?
3. Would it travel as fast as a Ferrari? A bullet? A shooting star?
4. How much damage would it do to your car if an ejected bomb hit it?

View of Sunset Crater from the west near Sugarloaf Mountain. Photo was taken by David Gonzales

Using math to find the velocity of bombs ejected from the Sunset eruption

Sunset Crater cone dimensions: The Sunset eruption is unusual because the volume of volcanic products (about 0.7 cubic miles, 3 cubic km) is large for a Strombolian event, the air fall dispersal was large, and the discharge rate for magma was high.

Strombolian eruptions are relatively low-level volcanic eruptions, named after the Italian volcano named Stromboli, where such eruptions consist of ejection of tephra to altitudes of tens to hundreds of meters. They are small to medium in volume, with sporadic violence.

In the last eruption of Sunset Crater in 1064 and 1065 A.D. (Duffield, 1997), fragments of volcanic rock and erupted magma were ejected into the area surrounding the cone.

Before you begin this exercise, it might be useful to visit the Visual Exercises and Component using Stromboli program at:

http://www.swisseduc.ch/stromboli/volcano/simulation/index-en.html

The goal of this study is to estimate the initial and terminal velocity of large volcanic bombs ejected from Sunset Crater.

Information and Assumptions

- Sunset Crater cone dimensions: the cone is about 1,000 feet (300 m) high and 1 mile (1.6 km) in base diameter
- Angle of ejection: $\alpha =$ 45 degrees
- The maximum distance that large volcanic bombs travel is 800 meters.

Notation:

- v_0 = initial velocity
- z_0 = height of the volcano

- t_{max} = the maximum time a bomb travels
- u = horizontal distance a bomb travels at time *t* (see formula 2 below.

The formulas below are only valid when the angle of ejection α = 45 degrees

Formula 1

The height (in meters) at time t is t $z = z_0 + \frac{v_0}{\sqrt{2}}t - \frac{1}{2}gt^2$ and $g = 9.8 m/s^2$.

Formula 2

The horizontal distance at time t is $u = \frac{v_0}{\sqrt{2}}t$.

Formula 3

The velocity at time *t* is $V = \sqrt{v_x^2 + v_y^2}$, where $v_x = \frac{v_0}{\sqrt{2}}$ and $v_y = \frac{v_0}{\sqrt{2}} - gt$.

Problem: To find t_{max} , the maximum time a bomb travels, v_0, the initial velocity of the bomb, and *V*, the total velocity at impact

Question 1: What is the initial velocity? To answer this, follow the steps below. Use meters for height and distance.

A) What is the height and the distance when the volcanic bombs hit the ground? (These are the values of *z* and *u* when $t = t_{max}$)

Answer: z = 0 and u = 800

B) Re-write formula 1 for *z* by substituting z_0= 300 and $g = 9.8 m/s^2$.

Answer: $z = 300 + \frac{v_0}{\sqrt{2}}t - 4.9t^2$

C) Use formula 2 and your answer to step A to write an equation for t_{max} in terms of v_0.

Answer: $t_{max} = \frac{800\sqrt{2}}{v_0}$

D) Substitute your answers from steps A, B, and C in the equation for *z* when $t = t_{max}$ and solve for v_0.

Answer: $v_0 = 75.5 m/s$

Question 2: What is t_{max}? (Use formula 2.)

Answer: t_{max} = 14.98

Question 3: What is *V*, the total velocity at impact? (Use formula 3.)

Answer: $V = 107.6 m/s$

Question 4: Steve builds a summer home just outside of Flagstaff, but only 2000 meters from Sunset Crater. Suppose there is another eruption from the crater. What will the initial velocity of a bomb need to be to destroy Steve's new vacation home?

Answer: v_0 = 130.6 *m/s*

Follow-up Questions

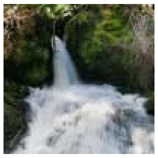
1. Describe the concepts from geology that were used in this study.

2. List the math you used to answer the questions above.

3. Do the equations you used give accurate estimates of speed and distance?

4. How do scientists find equations like this?

C. Stream Flow   Instructor Sheets



2 **Level 2   Snow Melt & Stream Flow in the Animas River   KM#3**

Brief Description: This study explores a simple way to predict streamflow, or the amount of water that flows through a stream. After an introduction, there are 3 parts, predicting temperature, precipitation (rain and snow), then the volume of water over the watershed is computed and used to predict streamflow. Then, given the average amount of precipitation, the volume of water contributed to the watershed is determined. This information is used to find the amount of water that goes into the stream, or the annual streamflow.

Mathematical Content: Area, Volume, Unit Conversion, linear equations. Kéyah Math Student Worksheets
The link above will provide the module questions in a format to use without computer for Math Tools

Objectives:

-

The ***Kéyah Math***

Introduction

This is a study about finding the average amount of water that flows through the Animas River during the months of snowmelt. The river has its headwaters above Silverton, Colorado, and flows south to Farmington, New Mexico, where it empties into the San Juan River. This study is concerned with the Upper Animas from its headwaters to Durango. It illustrates (in a simplified way) how scientists can use historic data to predict how snow melt affects the amount of water that runs through a stream in a certain time period. The volume of water that flows through a stream at a designated point in a specified time period is called *streamflow*, it can be recorded using various units, for example, in cubic feet per second (cfs), or cubic meters per second or cubic feet per month, etc.

Basically, it is measured by volume per time increment. If you measure streamflow at some point on a stream as 500 cfs, this means that 500 cubic feet of water passes through that point every second. For example, the streamflow for the Mississippi River at Baton Rouge is 211,000 cfs whereas the streamflow for the Colorado River below the Laguna Dam (on the Arizona/California border) is 398 cfs. In regions where there is a lot of snow, streamflow is influenced in a major way during the time when snow is melting. Indeed, the water available for drinking, household use, or irrigation is dependent on the amount of snowfall the region receives in winter.

This is the case for Animas River region since its headwaters and a large part of its path to the San Juan is located in the high San Juan Mountains with elevations from 7,000 to over 14,000 feet. If you completed KM Study #1, "Estimating Average Stream Flow for the Animas River," you found the average streamflow for a year; however the streamflow significantly fluctuates during the year, particularly during early summer when the mountain snow melts.

Geologists are interested in streamflow because it affects amounts of sediment carried by streams (this is somewhat dependent on streamflow) and how this might change stream beds and land formation.

Warm-up Questions

Use questions for thought as a preliminary discussion about the topic.

1. What is the streamflow for a river or stream near you?
2. Does the streamflow vary from time to time?
3. Is this stream fed by snow melt at any time during the year?
4. If so, what time of year and what happens to the stream?
5. Is global warming affecting the amount of rain or snow you get each year?

Using Math to See How Streamflow is Affected by Snow Melt (Version 1)

The questions that follow will lead you to predicting the monthly streamflow for the Animas River at Durango during the period when the winter snow is melting. We will measure this in cubic feet per month, then convert to cubic feet per second, the most common units used for streamflow in this country.

Below, Table 1 gives average monthly temperatures, in degrees Fahrenheit, for the drainage basin for the Animas River. This was obtained by averaging temperatures at Silverton and Durango.

Table 1

Month	**Jan**	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
Temp	20	24	31	38	47	55	61	60	53	43	30	21

Question 1: Assume that there is no snow melt during the months when average temperature is freezing or below; during this period, snow accumulates.
From the data above, find for which months snow is accumulating.

Answer: January, February, March, November, December

The data in the Table 2 below gives the average monthly precipitation, including water from snow, in inches, over the Animas River watershed. This was obtained by averaging precipitation at Silverton and Durango.

Table 2

Month	**Jan**	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
Precip	1.72	1.73	2.13	1.63	1.58	1.14	2.44	2.80	2.34	2.20	1.74	1.92

Sources: Monthly weather averages for Silverton, CO from Weather.com and Monthly Climate Summary from the Western Regional Climate Center

Question 2: From the data above, compute the total water (in inches) from snow during the months of freezing temperatures.

Answer: 9.24 inches

Question 3: Convert your answer to #2 to feet (1 ft = 12 inches).

Answer: 0.77 ft

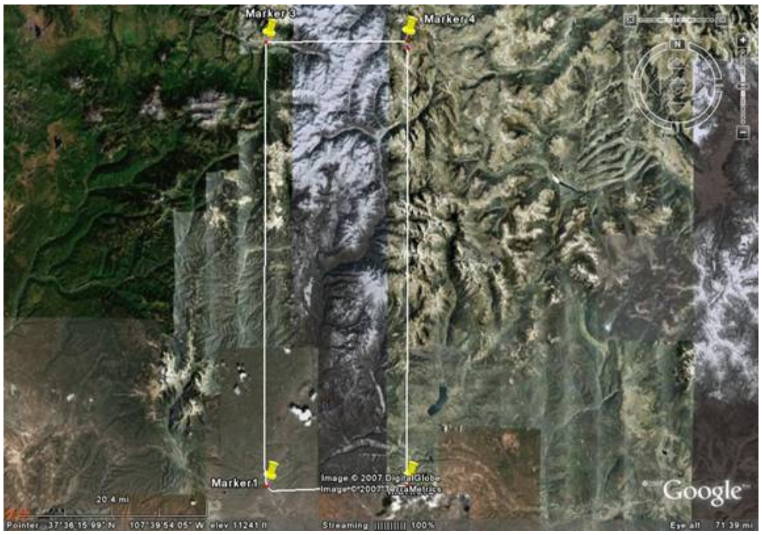
Question 4: The area of the drainage basin, or watershed, for the Animas from source to Durango is roughly 700 square miles.* **What is the area of the watershed in square feet?** (Using scientific notation, round to 2 decimal places.)

(1 mile = 5,280 feet, so 1 square mile = $(5280\ ft)^2$ = 5280^2 square feet)

Answer: $1.95 \times 10^{10}\ ft^2$

(Note: if you completed KM Study #1, you already have the answer to this question.)

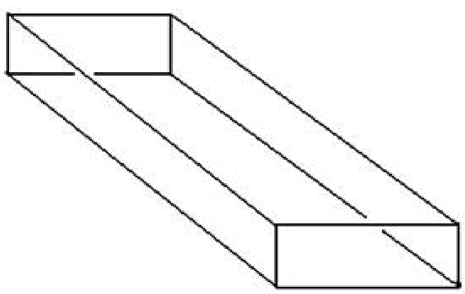
The picture below shows the approximate drainage basin for the Animas from source to Durango. Of course, the actual watershed is not rectangular but this shows the approximate region.



Source (The USGS web site Water Watch) shows the actual watershed is 692 square miles. You can check this out by going to the site and locating the dot for Durango in the southwest portion of the state.)

Question 5: Think of the drainage basin as a giant box with a rectangular base with area 700 square miles and height the amount of water from snow over the area (this is you answer to #3). (See figure left.)

The base of the figure is the rectangular watershed, and the height is the depth of water from snow



Now, find the total volume (in cubic feet—use your answers to #3 and 4) of water from snow that falls on the watershed during the freezing months. (Volume = Area of Base x Height; Using scientific notation, round to 2 decimal places.)

Answer: $1.50 \times 10^{10}\ ft^3$

Question 6: Assume that it takes 4 months for all the snow to melt after temperatures go above freezing. So, the Animas is getting water from both rain and snow melt during this period. From Table 2, convert the rainfall data to feet for each of the four months of snow melt. (Round to 2 decimal places.)

Answer: Apr 0.14 ft; May 0.13 ft; Jun 0.10 ft; Jul 0.20 ft.

Question 7: Use your four answers to #6 and your answer to #4 to compute the volume of rain over the watershed for each of the four months of snow melt. Your four answers will be in cubic feet. (Round to 2 decimal places.)

Answer: Apr 0.27×10^{10} ft^3 ; May 0.25×10^{10} ft^3 ; Jun 0.20×10^{10} ft^3 ; Jul 0.39×10^{10} ft^3

Question 8: As temperatures get warmer in the summer months, snow melts faster. Table 3 below gives the percentage of total snow that melts during each of the four months.

Now, refer to your answer to #5, and find the volume of snow that melts in each of the four months. Your four answers will be in cubic feet. (Round to 2 decimal places.)

Table 3

Month	1	2	3	4
%	2	38	56	4

Answer: Apr 0.03×10^{10} ft^3 ; May 0.57×10^{10} ft^3 ; Jun 0.84×10^{10} ft^3 ; Jul 0.06×10^{10} ft^3

Question 9: Next, find the total volume of water, rain from #7 + snow melt from #8, over the watershed for each of the four months. (Round to 2 decimal places.)

Answer: Apr 0.30×10^{10} ft^3 ; May 0.82×10^{10} ft^3 ; Jun 1.04×10^{10} ft^3 ; Jul 0.45×10^{10} ft^3

Question 10: Assume that only 74% of all water actually reaches the river bed (the rest is evaporated or drawn off for agricultural or domestic use). Find the streamflow for the Animas River for each of the four months of snow melt. (Round to 2 decimal places.)

Answer: Apr 0.22×10^{10} ft^3 ; May 0.61×10^{10} ft^3 ; Jun 0.77×10^{10} ft^3 ;

Question 11: .Convert your answers to #10 to cubic feet per second. (Round to nearest cubic foot.) (1 day = 24 hours, 1 hour = 60 minutes, and 1 minute = 60 seconds)

Answer: Jul 0.33×10^{10} ft^3

Question 12:

Answer: Apr 849 cfs; May 2277 cfs; Jun 2970 cfs; Jul 1232 cfs

Looking back at your answers

1. Review the method you used to find the streamflow.
2. Review the math you used to solve the problems above.
3. Do you think this is an accurate way to predict streamflow during the time of snow melt?
4. How are stream beds and erosion affected during times of high streamflow?
5. How would global warming affect streams, land formation, etc.?

6. Compare your answers to the data presented on the website,

http://pubs.usgs.gov/of/1992/ofr92-129/hcdn92/hcdn/ascii/monthlya/region14/09361500.amm.

D. How Big is the Earth?   Instructor Sheets

2 Level 2 Measuring the Size of the Earth KM#4

Brief Description: The goal of this study is to learn how Eratosthenes estimated the circumference of the earth. After explaining the geometry used by Eratosthenes to estimate the circumference of the Earth, students use this information to compute the radius and volume.
Mathematical Content: Geometry (radius and circumference of a circle; volume of a sphere); interior angles.

Kéyah Math Student Worksheets

The link above will provide the module questions in a format to use without computer for Math Tools

Objectives:

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The ***Kéyah Math***

Warm up question:

Use question for thought as a preliminary discussion about the topic.

Without leaving this country, how could you figure out how far it is all the way around the World?

Introduction

Around 250 BC, at noon on the day of the summer solstice (when the sun is at its highest point in the Northern Hemisphere) in Syrene, Egypt, sunlight filled the vertical shaft of a well; this indicates that the sun is directly overhead, so a vertical pole would cast no shadow. Eratosthenes, who lived in Alexandria, heard of this from a traveler. So on the same day, different year, he noticed that in Alexandria, some 800 kilometers (km) away, a vertical pole cast a shadow. From these observations, he made two deductions:

A. the earth is curved;

B. found the first estimate for the circumference of the Earth.

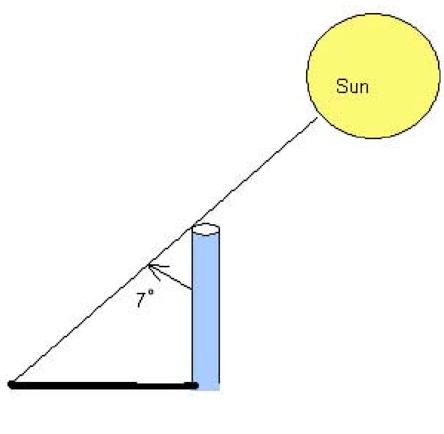
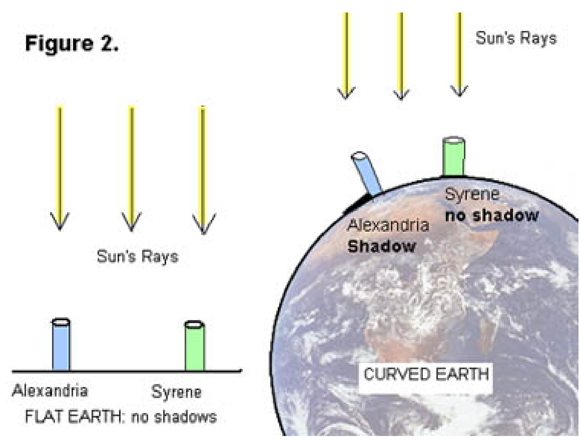


Figure 1

The Earth is Spherical

He measured the angle made by the pole and a line joining the tip of the shadow and the top of the pole (see Figure 1) and found the angle to be about 7°.

Figure 2.



Then he assumed that light rays from the sun to the Earth were essentially parallel since the sun was so far away and the Earth was so small relative to the sun. From this, and his observations in Alexandria and Syrene, he concluded that the Earth must be curved (see Figure 2), and therefore must be spherical.

Using Math to Find the Circumference of the Earth

Next, he used all this information to obtain the first nearly accurate estimation of the circumference of the Earth. Here's how: In the (not-to-scale) Figure 3

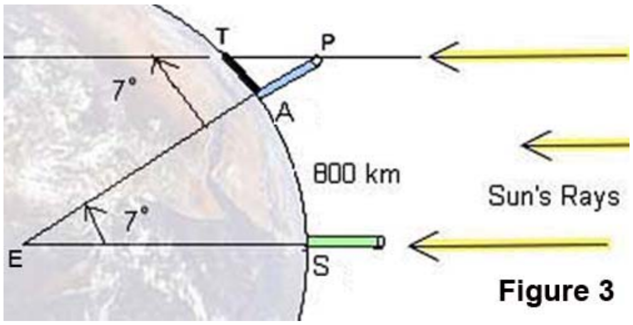


Figure 3

A denotes the base of the pole in Alexandria;

S the base of a pole in Syrene;

T the tip of the shadow cast by the pole in Alexandria;

P the top of the same pole;

E the center of the Earth.

Angle **APT** was measured to be 7°, so by Euclidean geometry interior angles $\angle APT$ and $\angle AES$ are equal, thus angle $\angle AE = 7^\circ$.

There are 360° in a complete circle, so the portion of the circumference of the Earth between **A** and **S** is

$\frac{7^\circ}{360^\circ}$, which is approximately $\frac{1}{50}$ (or, $\frac{360^\circ}{7^\circ}$ is approximately 50).

The distance from Alexandria to Syrene is 800 km, so he concluded that the circumference of the Earth must be $50 \times 800 = 40{,}000\ km$!

This estimate is very close to modern accurate measurements, so Eratosthenes gets credit for the first calculation of the size of the Earth.

We can get a slightly different answer if we compute more accurately:

$\frac{360}{7} = 51.4$, so if we multiply $51.4 \times 800 = 41{,}120\ km$

Questions

Notation:

Some formulas you'll need (r = radius of circle / sphere)

Circumference of a Circle: $C = 2\pi r$ Volume of a Sphere: $V = \frac{4}{3}\pi r^3$

Question 1: What is the radius of the Earth?

Use Eratosthenes' estimate for the circumference of the Earth to find its radius.

(Round your answer to 1 decimal place.)

Answer: $r = 6366.2\ \text{km}$

Question 2: What is the volume of the Earth?

Use your answer to Question 1 to compute the volume of the Earth. (Round your answer to 3 decimal places.)

Answer: $V = 1.081 \times 10^{12}\, km^3$

Here are some follow-up exercises:

1. Study Eratosthenes' method so you completely understand it.
2. There is a follow-up module to this one: it shows how to repeat this activity using locations in Arizona and suggests that you write a module using places in New Mexico, or elsewhere.

E. How Big is the Earth? Instructor Sheets

2 Level 2 Measuring the Size of the Earth from Arizona KM#5

Brief Description: This study uses the method of Eratosthenes with local information from Arizona to estimate the circumference of the earth. This estimate is then used to compute the Earth's radius and volume.

Mathematical Content: Geometry (radius and circumference of a circle; volume of a sphere); interior angles

Kéyah Math Student Worksheets

The link above right will provide the module questions in a format for students without computer for Math Tools.

Objectives:

-

The ***Kéyah Math***

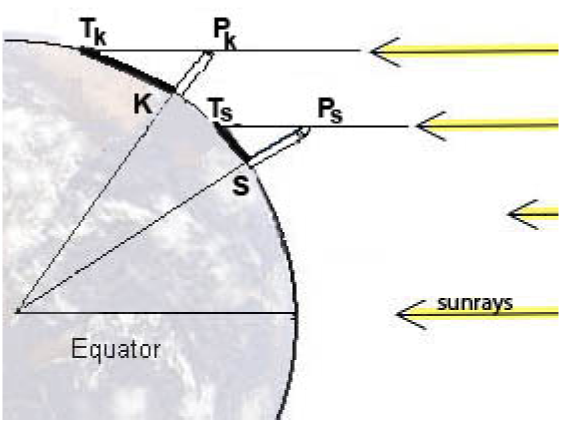
Introduction

This set of exercises gives you the opportunity to use the method of Eratosthenes locally (Arizona) to estimate the circumference of the Earth. Around 250 BC, at noon on the day of the summer solstice (when the sun is at its highest point in the Northern Hemisphere) in Syrene, Egypt, sunlight filled the vertical shaft of a well; this indicates that the sun is directly overhead, so a vertical pole would cast no shadow. Eratosthenes, who lived in Alexandria, heard of this from a traveler. So on the same day, different year, he noticed that in Alexandria, some 800 kilometers (km) away, a vertical pole cast a shadow. From these observations, he deduced that the earth is curved and he estimated the circumference: Module KM#4 provides a detailed description of this. (You can refer to that module for hints.)

This module is slightly different in that the locations we use are farther north than the ones Eratosthenes used, so the angles are different. We suppose the measurements here are taken at noon on the fall equinox, this is when the sun is directly above the equator on its path from summer to winter in the Northern Hemisphere. We use stations just south of Kaibito, Arizona and in Superior,Arizona (instead of Syrene and Alexandria, in Egypt). Kaibito is a small village on the Navajo Reservation in northern Arizona, almost due north of Superior, this will be one of the stations we use to complete these calculations. The other station is Superior, Arizona, a copper mining town about 60 miles east of Phoenix.

Using Math to Estimate the Circumference, Radius and Volume of the Earth

Vertical poles are erected at both stations and angles are measured similar to the way they were measured in by Eratosthenes. In the (not to scale) Figure 1:



S = the base of the pole in Superior, AZ;

K = the base of the pole in Kaibito, AZ;

T_S = the tip of the shadow cast by the pole in Superior;

T_K = the tip of the pole's shadow in Kaibito;

P_S = the top of the pole in Superior;

P_K = the top of the pole in Kaibito;

E = the center of the Earth

The measurements give these values:

$\angle SP_S T_S = 33.3°$ (try to measure this yourself on the next equinox);

$\angle KP_K T_K = 36.6°$ The direct distance from Superior to Kaibito is 367 km.

Question 1: What is the circumference of the Earth?

Hints: First label the figure with the measurements you have been given.

Can you determine the measure of $\angle KES$? Now follow the method of Eratosthenes to obtain your own estimation of the circumference of the Earth.

Answer: 40,778 km

Question 2: What is the radius of the Earth?

A. Compute the radius of the Earth using your estimate of the circumference.

(Round your answer to 1 decimal place.) Circumference of a circle: $C = 2\pi r$ where r is the radius.

Answer: $r = 6490 km$

B. Convert your answer to part A to meters (1 km = 1000 meters).

Answer: $r = 6490 \times 10^3\ m$

Question 3: What is the volume of the Earth?

Compute the volume of the Earth using your answer to Exercise 2 B.

(Round your answer to 3 decimal places.)

Volume of a Sphere: $V = \frac{4}{3}\pi r^3$ where r is the radius.

Answer: $V = 1.145 \times 10^{21} m^3$

Here are some follow-up exercises:

1. If you know some trigonometry you can use the information to determine the radius and then the circumference. Recall that the arc length $KS = r\theta$ where q is the measure of $\angle KES$ **in radians.** Convert degrees to radians and the solve for r. Now use this to determine the circumference and volume.

2. Look at a topographical map of New Mexico or Arizona and **choose two locations** to use to measure the circumference of the Earth. You'll need to know the latitudes and longitudes of your chosen locations in order to find the angles, and the places must be on approximately the same longitude (why?). You can also use the internet to find this information, go to www.topozone.com. Don't choose places too close together because this will decrease your accuracy. You'll need to estimate the direct distance between the locations—use the map scale or find it on the internet.

3. There are other variations on this exercise, for example, you can **change the time of year**, maybe to a solstice. You'll need the fact that the Earth's axis is tilted 23.5^{o} to the plane of the solar system. Or you can **change the location** to your favorite region of the World.

F. Earthquakes Instructor Sheets

2 Level 2 The Epicenter of an Southwestern Earthquake KM#6

Brief Description: This study examines the speed of P and S waves emitted from an earthquake, Equations relating distance traveled and speed are written, then the equations are used to find the epicenter as the intersection of three circles, each centered at a seismic station.
Mathematical Content: Basic algebra; equation of a line; radius of a circle. Kéyah Math Student Worksheets
The link above right will provide the module questions in a format for students without computer for Math Tools.

Objectives:

-

The ***Kéyah Math***

This is a study illustrating how earthquakes are located using data received at seismic stations. In particular, a 3.1 magnitude earthquake is used in this study. You will learn how geologists found out where it occurred.

Warm-up Steps

Use questions for thought as a preliminary discussion about the topic.

1. Do you know of any recent earthquakes near where you live?
2. If so, was any damage done?
3. Where have some earthquakes that you know about occurred?
4. How do you think that scientist know where an earthquake happened?
5. Why is it important to know where an earthquake occurred?

Introduction

An **earthquake** occurs when rocks in the crust move. This movement releases energy that is transmitted outward as **seismic waves**. **Seismic stations** have instruments that can sense and record seismic waves, even those from earthquakes that occur thousands of miles away from the station. Earthquakes generate several different types of seismic waves, and these waves each travel at different speeds through rock. The fastest waves (traveling at about 6 km/sec through most crustal rocks) are the first to arrive at seismic stations and are called primary (**P**) waves. The next fastest waves travel at about 3.5 km/sec, so they arrive at seismic stations later and have been termed secondary (**S**) waves. The farther these waves travel away from the source of the earthquake, the more the P waves will outpace the S waves.

The **epicenter** of an earthquake is a virtual point on the surface that is located directly above the source of the earthquake. The farther a given seismic station is from an epicenter, the longer the **time interval** between the arrival of the P waves and the arrival of the S waves. This time interval can be expressed mathematically as a function of the distance from the epicenter and the speeds of the P and S waves through rock. If we know what those two speeds are, and measure the time interval between the P and S arrivals at the seismic station, we can easily calculate how far the epicenter was from that seismic station.

The sequence of problems below will show you how this is done.

Using math to find the epicenter of an earthquake

Notation
The variables are:

t_p = number of seconds a P wave travels after the instant of the earthquake;

t_s = number of seconds a S wave travels after the instant of the earthquake;

d_p = distance (in km) P waves have traveled in *tp* seconds; and

d_s = distance (in km) S waves have traveled in *ts* seconds

Step 1: Write an equation that shows how far each type of wave traveled after *t* seconds.

Hint: Use the well-known formula, distance = rate x time,
to write the two equations that express distance traveled by the waves in terms of lapsed time. Your answer should be written in the form

$d_p = m_p t_p$, and $d_s = m_s t_s$, where m_p and m_s are the slopes of the lines.

Answer: $d_p = 6t_p$; $d_s = 3.5t_s$

Step 2:

A) Solve each of the two equations from Step 1 for times, t_s and t_p. Since the distance is the same for each wave, denote this by **D** (= d_p = d_s).

Answer: $t_p = d_p/6$; $ts = d_s/3.5$

B) Take the difference of the results from Part A; be sure your answer is positive. Denote this difference by the variable **U**.

Answer: $U = \frac{2.5}{21} D$

C) What does your answer to Part B represent?

Answer: Difference in arrival times for P- and S-waves

D) Solve the equation from Part B for distance **D**.

Answer: $D = \frac{21}{2.5} U$

The table below shows four seismic stations near New Mexico (represented by 3- and 4-letter codes) with latitude and longitude coordinates given in columns 2 and 3. Columns 4 and 5 indicate the times that P and S waves were received at the stations. The last three blank columns are for arrival time of P- and S-waves differences **U**, distances **D** from the stations to the epicenter of the earthquake, and scaled down distances for the map shown below.

Station	Arrival Time P-waves	Arrival Time S-waves	Time Diff U	Dist to Epicenter D	Scaled Distance for Map
1. TUC	**3:31:25**	**3:31:54**	30	252	2.45
2. ANMO	**3:31:28**	**3:32:00**	32	269	2.62
3. GDL2	**3:31:45**	**3:32:29**	44	370	3.6
4. LTX	**3:32:22**	**3:33:32**	70	588	5.72

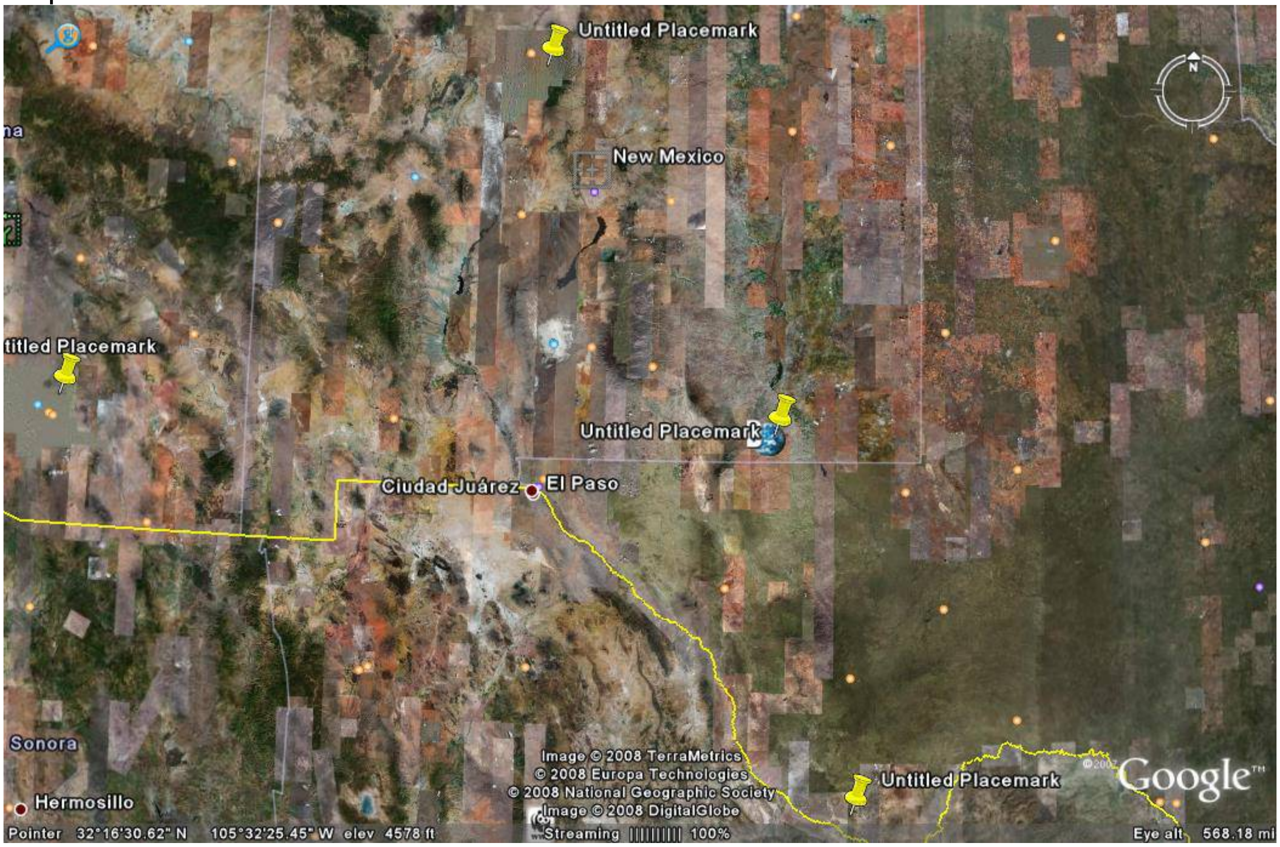
Step 3:

A) From the data given in the table above, compute the time differences in seconds that P and S waves were received at the stations. Complete the column for **U** in the table.

B) Use your answers to Part A and the equation from Step 2B to find the distances from the stations to the epicenter of the earthquake to fill in the column for **D**.

The distances you found in Step 3 can be used to find the epicenter of an earthquake. You know how far away the epicenter is from any one station but you don't know what direction. For example, if the epicenter is 1000 km away from the seismic station, you know it must be somewhere on the circle centered at the seismic station with radius 1000 km.

The map on this page is to be used to locate the epicenter. First, print this map.



Step 4.

This a Google Earth map. The four markers locate the four stations listed in the table. From the latitude and longitude coordinates in the table, locate and identify each station.

Answer: See Map Shown

Step 5.

Next, you will need to figure out the scale on the map.

A) Measure, in inches, the east-west distance on the map along the southern border of New Mexico from Texas to Arizona.

The real distance between these locations is 565.56 kilometers (Google Earth).

B) Now use this information to figure the scale for this map in km/inch.

Answer: A and B. On the Google map printed from my computer, the distance across the southern border of New Mexico is 5.5 inches. So the scale is 102.8km/inch. Note: this may vary with printing from different computers.

Step 6.

This is the final step!

A) Convert the distances **D** to fit the scale on the map; fill in the last column, **Scaled Distance for Map**, in the table with these data.

B) Use the distances you computed in the table for **U** and the scale for the map to draw a circle with center at each station and radius the scaled distance from that station to the epicenter.

C) Look at these circles and locate the epicenter.

D) What are the coordinates of the epicenter? What city or town is it near?

> Answer: B, C, D) Draw Circles with centers at Stations and radii shown in the table. Epicenter at 33.00N, 108.23W – just north of Silver City (See map)

Note: P- and S-wave travel is actually non-linear. For simplicity, we use a linear approximation in this study.

Follow-up Steps

1. Review the geology concepts used in this study.
2. Review the math that you used to find the epicenter.
3. Why is three the minimum number of stations that would be necessary to locate the earthquake?

G. Impact Processes Instructor Sheets

2 Level 2 Meteor Crater KM#7

Brief Description: This study examines Meteor Crater, Arizona, and the size of the meteorite that formed it. After an introduction to impact processes, students are guided through a sequence of steps that involve relatively simple formulas from physics to find the size of the meteorite. Specifically: a formula relating kinetic energy and diameter of the crater is used to find kinetic energy released on impact; a formula relating KE and mass is used to find the mass of the meteorite; a formula relating mass density and volume is used to find the volume of the meteorite; and finally, a formula relating spherical volume to radius is used to find the diameter of the meteorite.

Mathematical Content: Basic algebra, solving simple equations Kéyah Math Student Worksheets

The link above right will provide the module questions in a format for students without computer for Math Tools.

Objectives:

The ***Kéyah Math***

Meteor Crater is a distinct impression in the relatively featureless high steppe to the west of Winslow, Arizona. If you were, or are, seeing it for the first time, could you tell how it was formed?

It so happens that Meteor Crater is located in the vicinity of three different volcanic fields in the southwestern Colorado Plateau, so it is perhaps not surprising that some geologists initially thought it was formed by an explosive volcanic eruption. However, no volcanic rocks occur there. Instead, the Crater was littered with small balls of an iron-nickel alloy recognized immediately as having come from a meteoric source. A scientific debate, which extended from the late 19th into the early 20th Century, was finally resolved in favor of an origin brought about by the impact of an iron-nickel *meteorite*.

The largest impacts in Earth's past would have released tremendous energy, and blasted enough material into the upper atmosphere to at least temporarily change climates. Such catastrophes are thought to have triggered or at least contributed to mass extinctions of life on Earth, such as the extinction of the dinosaurs about 65 million years ago. Another impact like that would pose a direct threat to the future of humanity and all other life on Earth, so it is in our best interest to understand the processes and the effects of impact cratering as thoroughly as we can!

An important question that geologists would like to answer is, "What was the size of the meteorite that formed Meteor Crater?" This is the problem that we will answer in this exercise.

Warm-up Experiment (Optional)

Warm-up Questions

Use question for thought as a preliminary discussion about the topic.

1. Have you ever seen a crater that was formed by a meteor impact?
2. After looking a pictures of Meteor Crater, how wide and how deep do you think it is?
3. How big would the meteor have been to form this size crater?
4. How fast would it have been travelling?

Method of Attacking the Problem

Think about factors that would determine how "large" a crater is formed. In part, this would depend on geological conditions specific to the impact site, such as the mechanical properties of soils and rocks. However, one might also assume that kinetic energy, KE, of the meteorite is an important factor: the more energy delivered upon impact, the "bigger" the crater that is excavated.

There is a mathematical relationship between KE and crater diameter, we can measure the diameter of Meteor Crater and calculate the KE, and then the *mass*, of the meteorite. From the mass, we can then calculate the "size" (more precisely the *volume*) of the meteor, because volume and mass are related by *density*, and we have actual fragments of the meteorite on which density has been measured. So, here's an outline of the four-step approach we'll take:

- Use a formula relating KE and diameter of the crater to find KE released on impact
- Use a formula relating KE and mass to find the mass of the meteorite
- Use a formula relating mass density and volume to find the volume of the meteorite

Use a formula relating volume to radius to find the diameter of the meteorite

Important information you will need for Questions 1 and 5:
The diameter of meteor crater is 1200 meters

Question 1: How much energy was released when the meteorite formed the crater?

Read the information below for help with this question.

We will give you a formula that relates kinetic energy, KE, and diameter D of the crater. (If you want to see how this equation was obtained or find the equation yourself, click here.)

$$KE = 2.499D^{3.250} x10^{6}$$

The diameter D of Meteor Crater is measured in meters, and the answer for KE is in Joules. (As a rough guide, 1 joule is the amount of energy required to lift a one kilogram object up to a height of about 10 centimetres on the surface of the Earth by the most efficient method.)

Use this equation to find the kinetic energy released from the meteorite that formed meteor crater.

Answer: KE = 2.542×10^{16} Joules

Question 2: What was the mass of the meteorite?

You can answer this question using two additional pieces of information.

Information

The equation relating mass to kinetic energy: $KE = 0.5Ms^2$

where M is the mass (in kilograms, kg) of the meteorite, and **s** is its velocity (in meters per second).

An estimate for *s*, the velocity of the meteor. How fast do meteors typically travel? The average is about *20,000 meters/sec*, so use that value for s

Answer: $M = 1.271 \times 10^8$ kg

Question 3: How big was the meteorite, that is, what was its volume?

Again, you can answer this question by using two more bits of information and your previous answers

Information

The equation relating density, mass, and volume: $\rho = \frac{M}{V}$ where ρ (the greek letter "rho") is the density (in kilograms per cubic meter), M is the mass, and V is the volume (in cubic meters).

Fragments found at the site are iron-nickel, so the meteorite is assumed to be iron-nickel with density $\rho = 7800 kg / meter^3$.

Answer: V = 16295 cubic meters

Question 4: What was the diameter of the meteorite?

Notation:

Assuming that the meteorite was spherical, you'll need the formula for the volume V of a sphere:

$V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere (in meters).

Answer: diameter = 31.4 meters

Question 5: How many times larger was the diameter of the crater than the diameter of the meteorite?

Answer: 38.2 times larger

Follow Up Questions:

1. Review the concepts from geology that were used in this study.
2. Review the math you used to answer the questions above.
3. Do you think the equations you used give accurate estimates of the size of the meteor?
4. How do scientists find equations like this?

H. Age of the Earth — Instructor Sheets

2 Level 2+ Ages of Rocks and the Earth KM#8

Brief Description: This study explores methods for dating rocks using radioactive decay to find the age of rock from the San Juan Mountains and the age of the Earth. A basic decay equation for the Rubidium-Strontium isotope system is given and applied to date rock samples from Electra Lake, Southwest Colorado. Then this equation is then used to date a meteorite containing rubidium and strontium to estimate the age of the Earth.

Mathematical Content: Basic algebra; exponential equations; logarithms Kéyah Math Student Worksheets

The link above right will provide the module questions in a format for students without computer for Math Tools.

Objectives:

The ***Kéyah Math***

Introduction

How can we tell how old the Earth is?

Certain natural phenomena or processes, such as Earth's year-long solar orbit, and the resulting annual climatic variations that govern the growth of tree rings, can be used as "natural clocks."

If we can find and date a rock that we know has been around since the Earth formed, we can measure the age of the Earth. Can we find in rocks a natural clock that has been operating since they formed? It was discovered that some chemical elements, notably uranium and thorium, are strongly radioactive. These elements occur naturally in nearly all rocks, and they account for the radioactivity you could observe with a Geiger counter.

The radioactive decay process can be described simply as the transformation of an unstable radioactive atom (called the parent) to a new atom (called the daughter) that may differ in atomic number, atomic mass, or both. The transformation occurs either by loss of particles from, or addition of particles to, the parent nucleus.

In some parent-daughter pairs, the daughter is still radioactive and subject to further decay to a new daughter. In other cases, decay yields a daughter that is non-radioactive (stable) and will remain unchanged for the rest of time. The time interval it takes for the parent atoms to decay by half is always the same, no matter how much of the parent element remains. This constant length of time is called the ***half-life***.

How does radioactive decay serve as a "natural clock"?

Some common rocks are weakly radioactive. Numerous chemical analyses of crustal rocks have revealed that radioactive isotopes of elements such as uranium, thorium, potassium, and rubidium occur naturally in these rocks and account for their radioactivity. The precise half-lives of these isotopes have been measured experimentally.

These radioactive isotopes and their half-lives can be used as our natural clock, i.e., we can find out how old certain rocks are from this information.

Part 1. Using Math to Find the Age of Rock in Southwest Colorado

In this section, we will guide you through the process of finding the age of a sample of gabbro found at Electra Lake, just north of Durango in southwest Colorado. Following this, you can repeat these steps to estimate the age of the Earth.

Rubidium-Strontium Dating

Rubidium (^{87}Rb) decays to strontium (^{87}Sr) and because the half-life is so long, it is used by geologists to find the age of very old rock. The isotopes that are used for dating are ^{87}Rb, ^{87}Sr, and ^{86}Sr. ^{87}Rb decays to ^{87}Sr; ^{86}Sr is not a product of decay but is used as a reference isotope. This isotope system can be used as a natural clock to determine the age of many old rocks. This method is called Rubidium-Strontium dating by geologists.

The decay constant λ

Below is a table of the parent-daughter pair (or isotope system) that we will use in radiometric dating of the Electra Lake gabbro and the Earth. The half-life is given in million (10^6) years.

Isotope System		*Half-life T*
Parent isotope (symbol)	*Daughter isotope (symbol)*	*(in million years)*
Rubidium-87 (^{87}Rb)	Strontium-87 (^{87}Sr)	4.88×10^4

The decay constant, usually denoted by Greek letter lambda, λ, is specific to an isotope system. It can be determined experimentally or by using the half-life, and is equivalent to the fraction of atoms that decays per some interval of time. For the Rb-Sr system, $\lambda = 1.42 \times 10^{-5}$.

The Decay Equation

Rocks that contain ^{87}Rb also contain initial amounts of ^{87}Sr, so when comparing the relative amounts of ^{87}Rb and ^{87}Sr, the amount of ^{87}Sr present initially must be accounted for. Also, a certain amount of ^{86}Sr is present that is not a product of Rb decay, this is stable.
So when counting the amount of the daughter ^{87}Sr present now, the amount of ^{87}Sr present initially, $^{87}Sr_{initial}$, must be considered. Also, the amount of stable ^{86}Sr present must be accounted for. At time t = now, the ratio of both $^{87}Sr_{now}$ and $^{87}Rb_{now}$ to ^{86}Sr can be measured; hence the equation that can be used for the dating process is

$$\frac{^{87}Sr_{now}}{^{86}Sr} = \frac{^{87}Sr_{initial}}{^{86}Sr} + \frac{^{87}Rb_{now}}{^{86}Sr}(e^{\lambda t} - 1).$$

This equation has the form of a linear equation $y = b + mx$, where

$y = \frac{^{87}Sr_{now}}{^{86}Sr}$, $b = \frac{^{87}Sr_{initial}}{^{86}Sr}$, $x = \frac{^{87}Rb_{now}}{^{86}Sr}$, and slope $m = (e^{\lambda t} - 1).$

Now, Rb-Sr dating of a rock incorporates the following procedure that is followed below:

- Rock samples are collected from a site
- The ratios x and y are measured in each sample with a mass spectrometer
- The results are plotted on an (x, y)-axis system (this plot is called an ***isochron diagram***)
- The line of best fit--the line that comes closest to all the points, called the ***regression line***-- for the plot is found using the linear regression applet
- The slope of this line is set equal to m = $(e^{\lambda t} - 1)$
- The value of *t* is found from this equation—this is the age of the rock!

(Reference: Looking into the Earth; Musset & Khan; Cambridge University Press 2000)

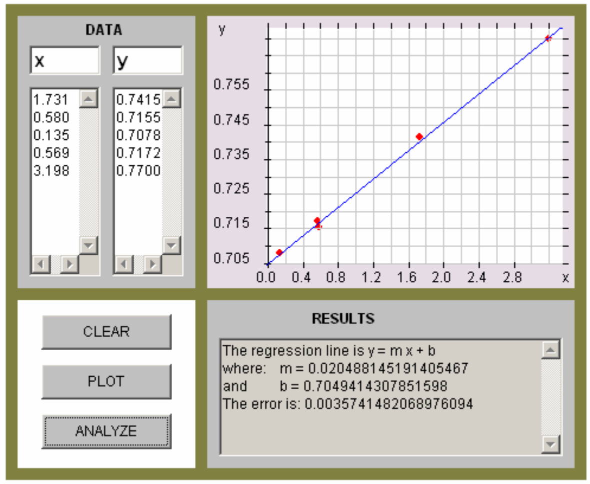
Information: Five samples of gabbro were collected near Electra Lake in southwest Colorado. Ratios x and y were measured by a mass spectrometer and are listed in the table below.

(Reference: Precambrian Rb-Sr Chronology in the Needle Mountains, Southwestern Colorado; Bickford, Wetherill, Barker, Lee-Hu; J. Geophysical Research, Vol. 74, No. 6, 1969)

X **Y** The graph shown below indicates the points plotted, the regression line

1.731	**0.7415**
0.580	**0.7155**
0.135	**0.7078**
0.569	**0.7172**
3.198	**0.7700**

and its equation. (This graph is from the linear regression applet.)



The equation of the regression line is

Y = 0.7049 + 0.0205x,

its slope is m = 0.0205.

Question 1: What is the age of the gabbro from Electra Lake?

(Hint: Equate the value for *m* to the slope in the decay equation and solve for *t*. This can be done algebraically using logarithms or by using the Plot-Solve applet.)

For a worked solution to this question, click here.

Answer: The gabbro is, approximately, 1,429 million years old.

Part 2. Using Math to Find the Age of the Earth

Here you will use Rubidium-Strontium decay to date a meteorite samples. Assuming that samples, the Earth, and the entire solar system were formed at approximately the same time, this should give us a good approximation to the age of the Earth.

Meteorites	87Rb/^{86}Sr	87Sr/^{86}Sr
Modoc	**0.86**	**0.757**
Homestead	**0.80**	**0.751**
Bruderheim	**0.72**	**0.747**
Kyushu	**0.60**	**0.739**
Buth Furnace	**0.09**	**0.706**

From five meteorite samples, a mass spectrometer measures the ratios of $^{87}Sr_{now}$ to ^{86}Sr and $^{87}Rb_{now}$ to ^{86}Sr; the data are listed in the table.

(Reference: Rubidium-Strontium Dating of Meteorites; http://hyperphysics.phy-astr.gsu.edu/hbase/nuclear/meteorrbsr.html)

Question 2: How old is the Earth?

In order to answer this question, you can repeat the procedure given in the dating of the gabbro; specifically:

- Plot the data on an (x, y)-axis system to create the isochron diagram
- Use linear regression to find the slope of the line of best fit
- Set the slope equal to $(e^{\lambda t} - 1)$
- Solve for *t*.

Answer: The earth is approximately 4.435 billion years old.

(You can solve this either algebraically or graphically—ask your Instructor. You may be provided a print-out of a graph that can be used to find the answer to this question, or you can use a graphing calculator, or go to the Kéyah Math or Earth Math website and use the Plot/Solve applet.)

VII. LEVEL 3 MODULES:

Algebra with functions; evaluating algebraic functions; solving equations; graphing

I. How Big is the Earth? Instructor Sheets

3 Level 3 Mass & Density of the Earth KM#9

Brief Description: This study is a follow-up to the size of the Earth modules. It uses estimates of the radius and volume of the Earth to compute its mass and density . Methods of Newton using the Law of Gravitational Attraction and density equations are used.
Mathematical Content: Geometry (radius and circumference of a circle; volume of a sphere); interior angles; Newton's Law of gravitational attraction; formulas for density and force; solving equations involving kinetic energy, mass and volume Kéyah Math Student Worksheets
The link above right will provide the module questions in a format for students without computer for Math Tools.

Objectives:

The ***Kéyah Math***
The goal of this study is to learn how the famous physicist, Sir Isaac Newton, computed the mass of the Earth, and then use this to compute its density.

Basic Concepts:

* Any object in the Universe attracts any other object.
* The force that moves objects toward each other is called ***gravity***.
* The ***mass*** of an object is a fundamental property of the object with measured in ***kilograms (kg)***.
* Even though we often use weight and mass interchangeably in everyday language, the weight of an object depends on the force of gravity. If you went to the moon, your mass would not change but your weight would be much less. The weight of an object is the force of gravity on the object; this can be described by the equation **w = mg** where ***m*** is the mass of the object and ***g*** is the acceleration due to gravity.
* The ***density*** of an object is the mass per unit volume.

In the year 1680, Sir Isaac Newton discovered the famous equation known as the Law of Gravitational Attraction on two objects. You will use this result, together with another result also due to Newton, to compute the mass of the Earth.
Notation:

Newton 's Law of Gravitational Attraction, $F = G\frac{m_1 m_2}{r^2}$, describes the force with which two objects attract each other.

- ***G*** is called the universal gravitational constant and its value is known
- m_1 and m_2 denote the masses of the two objects
- ***r*** is the distance between the objects.

For an object with mass m on the surface of the earth, this equation becomes

Equation 1: $F = m\frac{GM_E}{R_E^2}$ where M_E is the mass of the earth and R_E is the radius of the earth.
Newton's Second Law of Motion, *F* = mass x acceleration describes the relationship between force, mass and acceleration. On the surface of the earth the acceleration of gravity is about $g = 9.8\ meters/s^2$ **and we have Equation 2: *F* = *mg***.

Since equations 1 and 2 describe the same force, we have $m\frac{GM_E}{R_E^2} = F_{gravity} = mg$,

which simplifies to our basic equation Basic Equation: $\frac{GM_E}{R_E^2} = g$.

Thought questions

- If the distance between two objects increases does the force of attraction increase or decrease?
- Which object will have greater acceleration due to gravity, one of very large mass or one of very small mass?

You will use the basic equation $\frac{GM_E}{R_E^2} = g$ to estimate the mass, volume and density of the earth.

Information:

- **The average radius of the Earth is 6.38 x 10^6** meters
- The **universal gravitational constant** *G* is $G = 6.672 \times 10^{-8} meters^3/(Mg \cdot s^2)$

where Mg is megagrams (1 Mg = 1000 kilograms) and s is seconds.

- The acceleration of gravity at the surface of the Earth is about $g = 9.8\ meters/s^2$
- The volume of a sphere of radius *r* is given by the formula $V = \frac{4}{3}\pi r^3$
- $\text{Density} = \frac{\text{mass}}{\text{volume}}$

The answers to the following questions will use Newton's methods to determine the mass and density of the Earth.

Question 1: What is the mass of the Earth?

To answer this question, follow these steps:

- Substitute the values given above in the equation $\frac{GM_E}{R_E^2} = g$.

Solve the equation for ME, the mass of the Earth.

Answer: $5.936 \times 10^{24} kg$

Question 2 What is the volume of the Earth?

To answer this question, assume the Earth is a sphere with radius RE. Use the appropriate information given above.

Answer: $V = 1.0878 \times 10^{21} cubic\ meters$

Question 3 What is the density of the Earth?

Answer: $density = 5.457 \times 10^3\ kg/m^3$

Here are some follow-up questions to think about:

1. How do you think Newton discovered his Law of Gravitational Attraction?
2. Do you think this is an accurate method for computing the density of the Earth?
3. Do you know of modern, more high-tech techniques that might give a more accurate result?
4. How do you think geologists might measure the density of rocks?
5. Why do you think it's important to know the density of the Earth, or rocks that are part of the Earth?
6. What do you think the Earth is composed of to give it the density that you calculated?

J. Layers of the Earth — Instructor Sheets

3 Level 3 Layers of the Earth — KM#10

Brief Description: This study explores using graphs of travel times of seismic waves to discover a layering structure of the Earth. After an introduction, and using results for the diameter of the earth found in either module on the size of the Earth, the student is given information about the travel time of two seismic waves to a station a known distance from the epicenter of an earthquake. The student computes the velocity of the first seismic wave and the distance the second wave travels. From this information it is now possible to deduce the path of the second wave and conclude that the earth is layered. The depth of this layer and the radius of the inner barrier are also computed.
Mathematical Content: Basic algebra (distance, rate and time); geometry; Snell's law

Kéyah Math Student Worksheets

The link above right will provide the module questions in a format for students without computer for Math Tools.

Objectives:

The ***Kéyah Math***

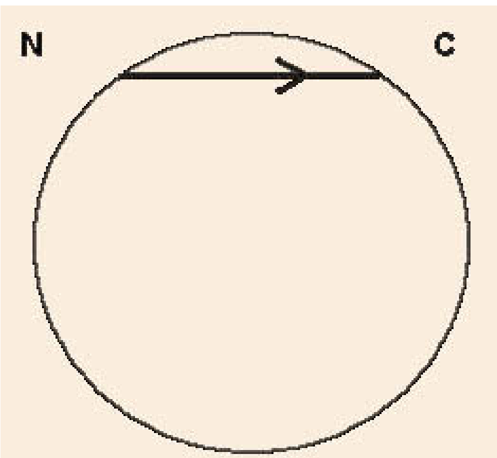


Figure 1

In the following exercises, **refer to Figure 1** (left), and **round all answers to 1 decimal place**.

Information:

You found the radius of the Earth to be 6,490 kilometers using the methods of Eratosthenes. Actually, the earth is not quite spherical, it bulges some at the equator, but the **average radius of the Earth is 6,371 km**, so that's what we'll use for this exploration.

An earthquake occurs at the indicated location labeled N on Figure 1. A seismograph located at a station labeled C which is 80 kilometers from N records the first seismic wave 12.3 seconds after the earthquake occurs.

Question 1: Using the known travel time and distance, compute the velocity of the seismic wave.

Answer: $6.5 km/s$

ASSUME: Since this is the first wave that reaches station C, we assume that this wave is the one that traveled in a straight line (shortest distance) from N to C. We assume that the distance, 80 km, from N to C along the surface of the Earth is approximately equal to the straight line distance from N to C.

A second wave arrives at station C 18.8 seconds after the earthquake occurs.

Question 2: Find the distance this second wave traveled. (Use the information about the second wave and the velocity you have computed.)

Answer: $d = 122.20 km$

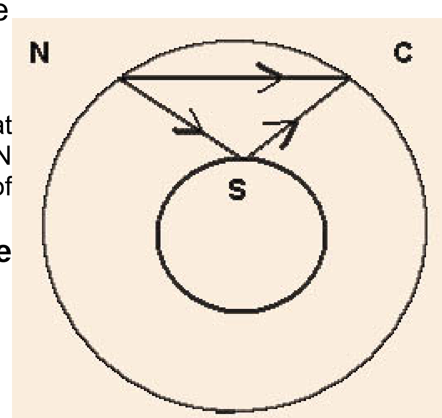


Figure 2

Question 3: Can you guess what path that this second wave traveled?

Answer: See picture in study

If you guessed that the wave must have bounced off some barrier inside the Earth, then you are correct! See Figure 2.

Next we will find the depth of this barrier. See Figure 3.

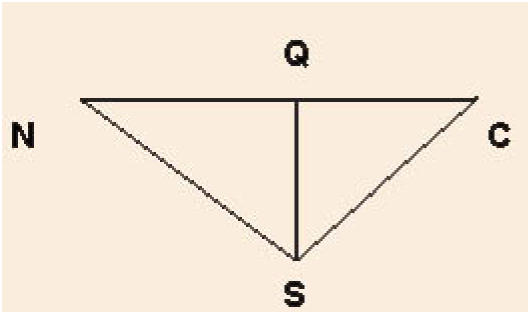


Figure 3

Information:

Laws of physics tell us:

- the **seismic wave bounces off the barrier at the point labeled S at the same angle that it hits the barrier.**

Also, from geometry, we know these facts:

- **the sides NS and SC have the same length;**
- **the line from S to Q divides the triangle NCS into two right triangles NQS and CQS that are the same size.**

Question 4: Compute the depth SQ of the barrier. (Use the information together with your previous answers to compute SQ, the depth of the barrier).

ASSUME: Here again, we assume that the straight line from N to C is the same as the very slightly curved line from N to C, so the length of QS is the same as the very slightly longer distance from S to the surface of the Earth.)

Answer: $SQ = 46.2\ km$

Question 5: Compute the radius of the interior barrier.

ASSUME: The Earth and this newly discovered interior "barrier" are both sphericalThis is one way to confirm that the Earth is layered, i.e., there's some inner core that's thick enough to reflect seismic waves!

Answer: $R_I = 6324\ km.$

K. How Big is the Earth? Instructor Sheets

3 Level 3 Size, Mass & Density of the Earth KM#11

Brief Description: The goal of this study is to learn how Eratosthenes estimated the circumference of the earth. After explaining the geometry used by Eratosthenes to estimate the circumference of the Earth, students use this information to compute the radius and volume. The students then apply the same method to local Arizona data and derive another estimate for the circumference and volume of the Earth. Finally, this estimate is then used to compute the Earth's density.
Mathematical Content: Geometry (radius and circumference of a circle; volume of a sphere); interior angles; Newton's Law of gravitational attraction; formulas for density and force; solving equations

Kéyah Math Student Worksheets
The link above right will provide the module questions in a format for students without computer for Math Tools.

Objectives:
This study has two parts. In the first part you will estimated the circumference, radius and volume of the Earth. In the second part you will determine the mass and density of the Earth. This module combines the material from KM#4 and KM#9.

Part 1: Estimating the Circumference of the Earth
The goal of this study is to learn how Eratosthenes made the first close estimate of the circumference of the Earth and then use his estimate to compute its radius and volume.

Warm up question:
Without leaving this country, how could you figure out how far it is all the way around the World?

Introduction
Around 250 BC, at noon on the day of the summer solstice (when the sun is at its highest point in the Northern Hemisphere) in Syrene, Egypt, sunlight filled the vertical shaft of a well; this indicates that the sun is directly overhead, so a vertical pole would cast no shadow. Eratosthenes, who lived in Alexandria, heard of this from a traveler. So on the same day, different year, he noticed that in Alexandria, some 800 kilometers (km) away, a vertical pole cast a shadow. From these observations, he made two deductions:

A. the earth is curved;

B. found the first estimate for the circumference of the Earth.

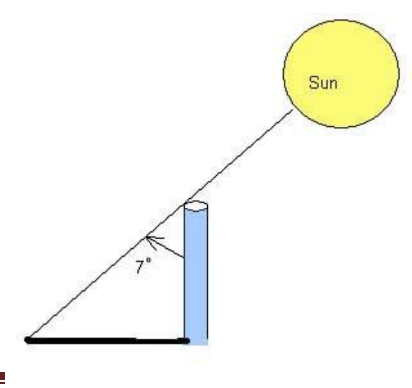
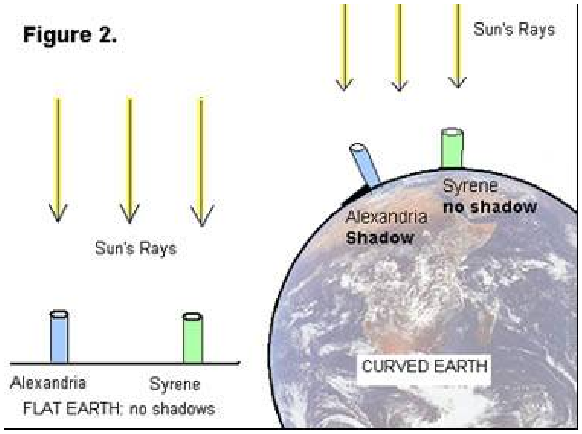


Figure 1

Figure 2.



The Earth is Spherical

He measured the angle made by the pole and a line joining the tip of the shadow and the top of the pole (see Figure 1) and found the angle to be about 7^o. Then he assumed that light rays from the sun to the Earth were essentially parallel since the sun was so far away and the Earth was so small relative to the sun. From this, and his observations in Alexandria and Syrene, he concluded that the Earth must be curved (see Figure 2), and therefore must be spherical.

Using Math to Find the Circumference of the Earth

Next, he used all this information to obtain the first nearly accurate estimation of the circumference of the Earth.

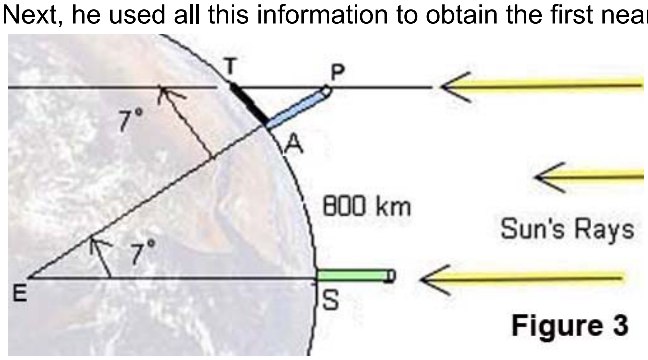


Figure 3

Here's how:

In the (not-to-scale) Figure 3:
A denotes the base of the pole in Alexandria;
S the base of a pole in Syrene;
T the tip of the shadow cast by the pole in Alexandria;
P the top of the same pole;
E the center of the Earth.

Angle **APT** was measured to be 7^o, so by Euclidean geometry interior angles $\angle APT$ and $\angle AES$ are equal, thus angle $\angle AE = 7^o$.

There are 360o in a complete circle, so the portion of the circumference of the Earth between **A** and **S** is $\frac{7^o}{360^o}$, which is approximately $\frac{1}{50}$ (or, $\frac{360^o}{7^o}$ is approximately 50).

The distance from Alexandria to Syrene is 800 km, so he concluded that the circumference of the Earth must be $50 \times 800 = 40{,}000\ km$!

This estimate is very close to modern accurate measurements, so Eratosthenes gets credit for the first calculation of the size of the Earth. We can get a slightly different answer if we compute more accurately: $\frac{360}{7} = 51.4$, so if we multiply $51.4 \times 800 = 41{,}120\ km$

Notation:
Some formulas you'll need (r = radius of circle / sphere)
Circumference of a Circle: **C =** $2\pi r$

Volume of a Sphere: $V = \frac{4}{3}\pi r^3$

Question 1: What is the radius of the Earth?
Use Eratosthenes' estimate for the circumference of the Earth to find its radius.
(Round your answer to 1 decimal place.)

Answer: $r = 6366.2\text{ km}$

Question 2: What is the volume of the Earth?
Use your answer to Question 1 to compute the volume of the Earth. (Round your answer to 3 decimal places.)

Answer: $V = 1.081 \times 10^{12}\ km^3$

Part 2: Determining the Mass and Density of the Earth

The goal of this study is to learn how the famous physicist, Sir Isaac Newton, computed the mass of the Earth, and then use this to compute its density.

Basic Concepts

- Any object in the Universe attracts any other object.
- The force that moves objects toward each other is called ***gravity***.
- The ***mass*** of an object is a fundamental property of the object with measured in ***kilograms (kg)***.
- Even though we often use weight and mass interchangeably in everyday language, the weight of an object depends on the force of gravity. If you went to the moon, your mass would not change but your weight would be much less. The weight of an object is the force of gravity on the object; this can be described by the equation

w = mg where *m* is the mass of the object and *g* is the acceleration due to gravity.

- The *density* of an object is the mass per unit volume.

In the year 1680, Sir Isaac Newton discovered the famous equation known as the Law of Gravitational Attraction on two objects. You will use this result, together with another result also due to Newton, to compute the mass of the Earth.

Notation:

Newton 's Law of Gravitational Attraction, $F = G\frac{m_1 m_2}{r^2}$, describes the force with which two objects attract each other.

- ***G*** is called the universal gravitational constant and its value is known
- m_1 and m_2 denote the masses of the two objects
- ***r*** is the distance between the objects.

For an object with mass *m* on the surface of the earth, this equation becomes
Equation 1:

$F = m\frac{GM_E}{R_E^2}$	where M_E is the mass of the earth and R_E is the radius of the earth.

Newton's Second Law of Motion, *F* = mass x acceleration describes the relationship between force, mass and acceleration. On the surface of the earth the acceleration of gravity is about $g = 9.8\ meters/s^2$ and we have **Equation 2: *F* = *mg***.

Since equations 1 and 2 describe the same force, we have $m\frac{GM_E}{R_E^2} = F_{gravity} = mg$,which simplifies to our basic equation

Basic Equation: $\frac{GM_E}{R_E^2} = g$.

Thought questions

- If the distance between two objects increases does the force of attraction increase or decrease?
- Which object will have greater acceleration due to gravity, one of very large mass or one of very small mass?

You will use the basic equation $\frac{GM_E}{R_E^2} = g$ to estimate the mass, volume and density of the earth.

Information:

- **The average radius of the Earth is 6.38 x 10^6** meters
- The **universal gravitational constant** *G* is $G = 6.672 \times 10^{-8} \, meters/(Mg \cdot s^2)$ where Mg is megagrams (1 Mg = 1000 kilograms) and s is seconds.
- The acceleration of gravity at the surface of the Earth is about $g = 9.8 \, meters/s^2$
- The volume of a sphere of radius *r* is given by the formula $V = \frac{4}{3}\pi r^3$
- $Density = \frac{mass}{volume}$

The answers to the following questions will use Newton's methods to determine the mass and density of the Earth.

Question 1: What is the mass of the Earth?

To answer this question, follow these steps:

- Substitute the values given above in the equation $\frac{GM_E}{R_E^2} = g$.
- Solve the equation for ME, the mass of the Earth.

  Answer: $5.936 \times 10^{24} kg$

Question 2: What is the volume of the Earth?

To answer this question, assume the Earth is a sphere with radius RE. Use the appropriate information given above.

Answer: $V = 1.0878 \times 10^{21} cubic\ meters$

Question 3: What is the density of the Earth?

Use your answers from the first two questions to answer this one.

Answer: $density = 5.457 \times 10^3 \, {kg}/{m^3}$

Here are some follow-up questions to think about:

1. How do you think Newton discovered his Law of Gravitational Attraction?
2. Do you think this is an accurate method for computing the density of the Earth?
3. Do you know of modern, more high-tech techniques that might give a more accurate result?
4. How do you think geologists might measure the density of rocks?
5. Why do you think it's important to know the density of the Earth, or rocks that are part of the Earth?
6. What do you think the Earth is composed of to give it the density that you calculated?

LEVEL 4 MODULES:

Pre-calculus; algebraic and exponential functions; evaluation; graphing; geometry

L. Age of the Earth — Instructor Sheets

4 Level 4 Geochronology in the San Juan Mountains KM#12

Brief Description: This study explores methods for dating rocks using radioactive decay to find the age of rock from the San Juan Mountains and the age of the Earth. A basic decay equation for the Rubidium-Strontium isotope system, is derived and applied to date rock samples from Electra Lake, Southwest Colorado. Then this equation is used to date a meteorite containing rubidium and strontium to estimate the age of the Earth.
Mathematical Content: Basic algebra; exponential equations; logarithms; linear regression.

Kéyah Math Student Worksheets

The link above right will provide the module questions in a format for students without computer for Math Tools.

Objectives:

The ***Kéyah Math***

Introduction
How can we tell how old the Earth is?

Certain natural phenomena or processes, such as Earth's year-long solar orbit, and the resulting annual climatic variations that govern the growth of tree rings, can be used as "natural clocks."If we can find and date a rock that we know has been around since the Earth formed, we can measure the age of the Earth. Can we find in rocks a natural clock that has been operating since they formed? It was discovered that some chemical elements, notably uranium and thorium, are strongly radioactive. These elements occur naturally in nearly all rocks, and they account for the radioactivity you could observe with a Geiger counter.

The radioactive decay process can be described simply as the transformation of an unstable radioactive atom (called the parent) to a new atom (called the daughter) that may differ in atomic number, atomic mass, or both. The transformation occurs either by loss of particles from, or addition of particles to, the parent nucleus.

In some parent-daughter pairs, the daughter is still radioactive and subject to further decay to a new daughter. In other cases, decay yields a daughter that is non-radioactive (stable) and will remain unchanged for the rest of time. The time interval it takes for the parent atoms to decay by half is always the same, no matter how much of the parent element remains. This constant length of time is called the ***half-life***.

How does radioactive decay serve as a "natural clock"?

Some common rocks are weakly radioactive. Numerous chemical analyses of crustal rocks have revealed that radioactive isotopes of elements such as uranium, thorium, potassium, and rubidium occur naturally in these rocks and account for their radioactivity. The precise half-lives of these isotopes have been measured experimentally.

These radioactive isotopes and their half-lives can be used as our natural clock, i.e., we can find out how old certain rocks are from this information.

Part 1. Using Math to Find the Age of Rock in Southwest Colorado

In this section, we will guide you through the process of finding the decay constant for a radioactive isotope in the basic decay equation, and then use this and another decay equation to find the age of a sample of gabbro found at Electra Lake, just north of Durango in southwest Colorado. Following this, you can repeat these steps to estimate the age of the Earth.

Rubidium-Strontium Dating

Rubidium (^{87}Rb) decays to strontium (^{87}Sr) and because the half-life is so long, it is used by geologists to find the age of very old rock. The isotopes that are used for dating are ^{87}Rb, ^{87}Sr, and ^{86}Sr. ^{87}Rb decays to ^{87}Sr; ^{86}Sr is not a product of decay but is used as a reference isotope. This isotope system can be used as a natural clock to determine the age of many old rocks. This method is called Rubidium-Strontium dating by geologists.

The *decay function* is $P(t) = P_0 e^{-\lambda t}$, where P(t) = number of atoms of the parent isotope at time t,

P_0 = P(0) = initial number of atoms of the parent isotope (at t = 0, when decay started),

λ = (Greek letter lambda) the decay constant specific to that parent isotope, which can be determined experimentally, and is equivalent to the fraction of atoms that decays per some interval of time.

We can use the basic decay equation and isotope half-life to find the decay constant for specific isotopes.

Finding the decay constant λ

Below is a table of the parent-daughter pair (or isotope system) that we will use in radiometric dating of the Electra Lake gabbro and the Earth. The half-life is given in million (10^6) years.

Isotope System		*Half-life T*
Parent isotope (symbol)	*Daughter isotope (symbol)*	*(in million years)*
Rubidium-87 (87 Rb)	Strontium-87 (87 Sr)	4.88 x 104

Question 1. What is the value of the decay constant for rubidium-strontium?

Solution to Question 1.

Use the basic decay equation P(t) = P_0e- λ t. When t = half-life for Rb-Sr = 4.88 x 10^4 million years,

then the initial number of atoms is reduced by half (from P_0 to $\frac{P_0}{2}$) , so denoting half-life by *h*, set *t* = *h* and solve $P(h) = \frac{P_0}{2}$ for l. $P(4.88\times10^4) = P(h) = \frac{P_0}{2} = P_0 e^{-\lambda h}$,

So now we solve $\frac{P_0}{2} = P_0 e^{-\lambda h}$ for λ, with *h* = 4.88 x 10^4. $\frac{P_0}{2} = P_0 e^{-\lambda(4.88\times10^4)}$

Cancel P_0 from each side, then take the natural logarithm of both sides,

$$\frac{1}{2} = e^{-\lambda(4.88\times10^4)} \qquad \ln\left(\frac{1}{2}\right) = \ln\left(e^{-\lambda(4.88\times10^4)}\right) = -\lambda(4.88\times10^4)$$

But $\ln\left(\frac{1}{2}\right) = \ln(1) - \ln(2) = 0 - \ln(2) = -\ln(2)$ So $-\ln(2) = -\lambda(4.88\times10^4)$, $\lambda = \frac{\ln 2}{4.88\times10^4} = 1.42\times10^{-5}$.

We have found the decay constant λ = 1.42 x10^{-5} for the Rb-Sr system. $\lambda = 1.42\times10^{-5}$

The decay constant λ = 1.42 x10^{-5} for the Rb-Sr system.

Question 2. Suppose that in an isotope system, the parent decays into the daughter, and there was no daughter atoms present initially. What is the ratio of daughter atoms to parent atoms at time *t*?

Solution to Question 2.

- First, solve the basic decay equation for P_0, $P_0 = \frac{P(t)}{e^{-\lambda t}} = P(t)e^{\lambda t}$
- Now let D(t) denote the number of daughter atoms at time t. The number of daughter atoms at time t is equal to the number of parent atoms present initially minus the number of parent atoms at time t, so

$$D(t) = P_0 - P(t) = P(t)e^{\lambda t} - P(t) = P(t)\left(e^{\lambda t} - 1\right)$$

$$\frac{D(t)}{P(t)} = e^{\lambda t} - 1$$

This solution to Question 2, $\frac{D(t)}{P(t)} = e^{\lambda t} - 1$, shows the desired ratio—***note that this is true for any such isotope system--***this equation is called the ***basic decay equation***.

Modification of the Basic Decay Equation for Rb-Sr Dating

First, solve the basic decay equation for the number of daughter atoms, *D(t)*.

$$D(t) = P(t)\left(e^{\lambda t} - 1\right)$$

Rocks that contain ^{87}Rb also contain initial amounts of ^{87}Sr, so when comparing the relative amounts of ^{87}Rb and ^{87}Sr, the amount of ^{87}Sr present initially must be accounted for. Also, a certain amount of ^{86}Sr is present that is not a product of Rb decay, this is stable.

So when counting the amount of the daughter ^{87}Sr, the basic decay equation must be modified to add in the amount present initially, ^{87}Srinitial, to the amount decayed at time t, ^{87}Srt. This gives the modified equation, $^{87}Sr_t = {}^{87}Sr_{initial} + {}^{87}Rb_t\left(e^{\lambda t} - 1\right)$.

At time t = now, the ratio of both ^{87}Srt and ^{87}Rbt to ^{86}Sr can be measured, so divide both sides of the above equation by ^{86}Sr (which is constant) to get $\frac{^{87}Sr_{now}}{^{86}Sr} = \frac{^{87}Sr_{initial}}{^{86}Sr} + \frac{^{87}Rb_{now}}{^{86}Sr}\left(e^{\lambda t} - 1\right)$.

This equation has the form of a linear equation y = b + mx, where

$y = \frac{^{87}Sr_{now}}{^{86}Sr}$, $b = \frac{^{87}Sr_{initial}}{^{86}Sr}$, $x = \frac{^{87}Rb_{now}}{^{86}Sr}$, and slope $m = \left(e^{\lambda t} - 1\right)$.

Now, Rb-Sr dating of a rock incorporates the following procedure that is followed below:

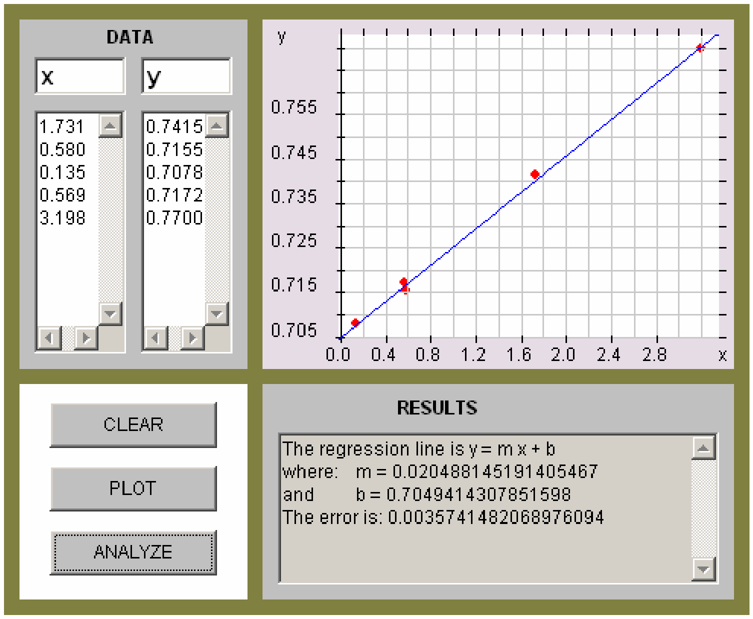
- Rock samples are collected from a site
- The ratios x and y are measured in each sample with a mass spectrometer
- The results are plotted on an (x, y)-axis system (this plot is called an ***isochron diagram***)
- The line of best fit for the plot is found using linear regression
- The slope of this line is set equal to m = $\left(e^{\lambda t} - 1\right)$
- The value of *t* is found from this equation—this is the age of the rock!

(Reference: Looking into the Earth; Musset & Khan; Cambridge University Press 2000)

Information: Five samples of gabbro were collected near Electra Lake in southwest Colorado. Ratios x and y were measured by a mass spectrometer and are listed in the table below.

X	Y
1.731	0.7415
0.580	0.7155
0.135	0.7078
0.569	0.7172
3.198	0.7700

The graph shown below indicates the points plotted, the regression line and its equation. (This graph is from the linear regression applet.)



(Reference: Precambrian Rb-Sr Chronology in the Needle Mountains, Southwestern Colorado; Bickford, Wetherill, Barker, Lee-Hu; J. Geophysical Research, Vol. 74, No. 6, 1969)

The equation of the regression line is Y = 0.7049 + 0.0205x, its slope is m = 0.0205.

Question 3. What is the age of the gabbro from Electra Lake?

Solution to Question 3

Solve the equation 0.0205 = $e^{\lambda t} - 1$, with λ = decay constant for Rb-Sr = 1.42 x10-5, for t.

We present two solutions, one is algebraic, the other is graphic using the plot-solve applet.

Algebraic solution.

Solve $0.0205 = e^{(1.42\times10^{-5})t} - 1 \qquad 1.0205 = e^{(1.42\times10^{-5})t}$

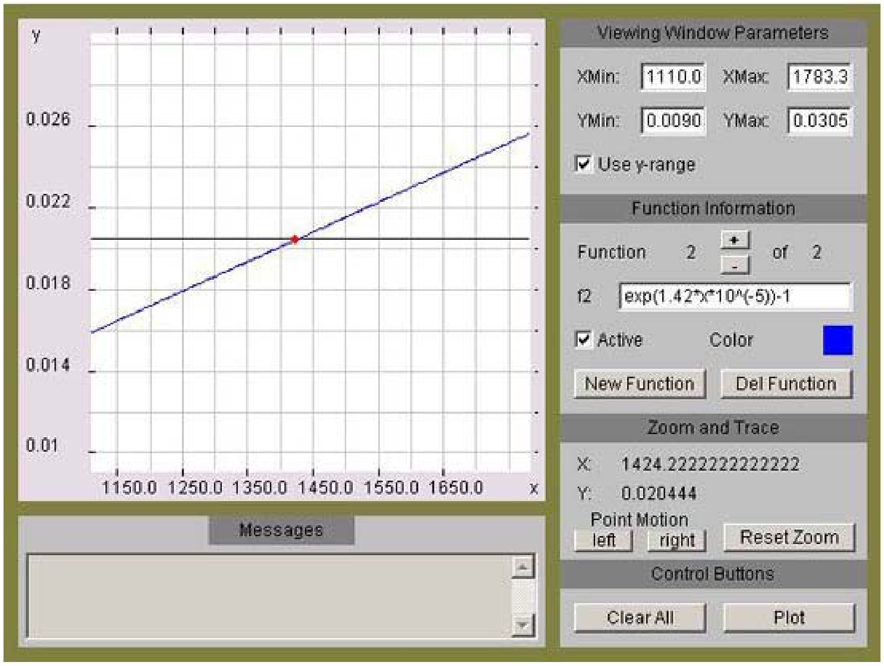
Take the natural logarithm of both sides and then solve for t,

$$\ln(1.0205) = \ln\left(e^{1.42\times10^{-5}t}\right) = 1.42\times10^{-5}t \qquad t = \frac{\ln(1.0205)}{1.42\times10^{-5}} = 1429$$

The gabbro is, approximately, 1,429 million years old.

Graphical Solution

To solve the equation $0.0205 = e^{(1.42\times10^{-5})t} - 1$, graph each side and determine the point of intersection.



(The graph shown to the left is copied from the Earth Math or Kéyah Math website Plot/Solve applet.)

The gabbro is, approximately, 1,429 million years old.

Part 2. Using Math to Find the Age of the Earth

Here you will use Rubidium-Strontium decay to date a meteorite samples. Assuming that samples, the Earth, and the entire solar system were formed at approximately the same time, this should give us a good approximation to the age of the Earth.

Meteorites	$^{87}Rb/^{86}Sr$	$^{87}Sr/^{86}Sr$
Modoc	0.86	0.757
Homestead	0.80	0.751
Bruderheim	0.72	0.747
Kyushu	0.60	0.739
Buth Furnace	0.09	0.706

From five meteorite samples, a mass spectrometer measures the ratios of $^{87}Sr_{now}$ to ^{86}Sr and $^{87}Rb_{now}$ to ^{86}Sr; the data are listed in the table to the left.

(Reference: Rubidium-Strontium Dating of Meteorites; http://hyperphysics.phy-astr.gsu.edu/hbase/nuclear/meteorrbsr.html)

Question. How old is the Earth?

In order to answer this question, you can repeat the procedure given in the dating of the gabbro; specifically:

- Plot the data on an (x, y)-axis system to create the isochron diagram
- Use linear regression to find the slope of the line of best fit

  Answer: m = 0.065
- Set the slope equal to $(e^{\lambda t} - 1)$
- Solve for *t*. Answer: t = 4435

  Earth is 4.435 billion years old

(You can solve this either algebraically or graphically—ask your Instructor. You may be provided a print-out of a graph that can be used to find the answer to this question, or you can use a graphing calculator, or go to the Kéyah Math or Earth Math website and use the Plot/Solve applet.)

M. Impact Processes Instructor Sheets

4 Level 4 Impact Processes at Meteor Crater (Advanced) KM#13

Brief Description: This study examines Meteor Crater, Arizona, and the size of the meteorite that formed it. After an extensive introduction to impact processes, students are guided through a sequence of steps that involve formulas from physics to find the size of the meteorite. Specifically: a formula relating kinetic energy and diameter of the crater is found using power regression on real data. This is used to find kinetic energy released on impact; a formula relating KE and mass is used to find the mass of the meteorite; a formula relating mass density and volume is used to find the volume of the meteorite; and finally, a formula relating spherical volume to radius is used to find the diameter of the meteorite.
Mathematical Content: Algebra, power and linear equations, linear regression, solving equations involving kinetic energy, mass and volume

Kéyah Math Student Worksheets

The link above right will provide the module questions in a format for students without computer for Math Tools.

Objectives:

The ***Kéyah Math***

What was the size of the meteorite that formed Meteor Crater?

Warm-up Questions

1. Have you ever seen a crater that was formed by a meteor impact?
2. After looking at pictures of Meteor Crater, how wide and how deep do you think it is?
3. How big would the meteor have been to form this size crater?
4. How fast would it have been travelling?

Understand the Problem:

A meteor falling toward the Earth is propelled by gravitational attraction. Because it is moving, the meteorite has an energy of movement or *kinetic energy (KE)*, which is described by the equation:

$KE = 0.5Ms^2$ **where M** is the mass of the meteorite and **s** is its velocity.

If the meteorite is accelerating downward, its KE must be increasing as the *square* of the velocity! If the meteorite is big enough, it will pass through the atmosphere without burning up completely. When it strikes the surface of the Earth, its velocity and KE go to zero in an instant, but the law of conservation of energy holds that the energy is not simply lost; it is transferred to the surroundings as heat, light, and work, sending out shock waves and excavating a crater far larger than the meteorite itself.

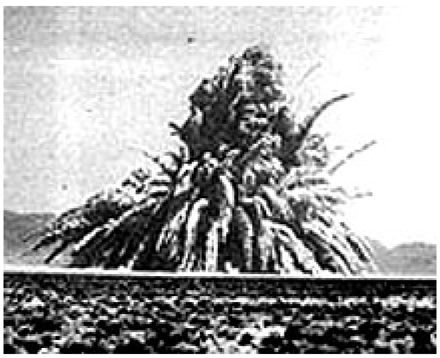
Consider the factors that would determine how "large" a crater is formed. In part, this would depend on geological conditions specific to the impact site, such as the mechanical properties of soils and rocks. However, one might also assume that KE of the meteorite is a more important factor: the more energy delivered upon impact, the "bigger" the crater that is excavated. (We will use *diameter* to represent crater "size," because as a crater is eroded away through time, its diameter changes far less than its depth.)

If KE is the most important (controlling) factor, and we can find a mathematical relationship between KE and crater diameter, we can take the dimensions of Meteor Crater and calculate the KE, and then the *mass*, of the offending meteorite. From the mass, we can then calculate the "size" (more precisely the *volume*) of the meteorite, because volume and mass are related by *density*, and we have actual fragments of the meteorite on which density has been measured.

Gather Data:

What data are available to help us solve this problem?

The impact that formed Meteor Crater is beyond history; evidence suggests that it occurred about 50,000 years ago. An impact of this size has not been observed on Earth in recorded time (and most would likely consider that a good thing!). But in the last century, for better or for worse, human beings have devised and experimented with a process of comparable destructive power: *nuclear explosions.* Until the advent of treaties restricting the practice, nations tested nuclear weapons by detonating them at or just beneath the surface.




Sedan, NV nuclear testing explosion and resulting crater. Image source http://rst.gsfc.nasa.gov/

In the United States, most nuclear weapons testing took place at the Nevada Test Site, in the Mojave and Great Basin Deserts of south-central Nevada, on the homelands of the Newe (Western Shoshone) people. The Newe had no say in the testing, and continue to work for restoration of these lands.

Nuclear explosions often excavated craters identical to those attributed to meteorite impacts. TheKE released in these blasts was known to the weapons designers, so here was a relationship between energy and crater size. This information was eventually made public, and planetary geoscientists made use of this relationship to estimate the KE needed to excavate impact craters of various sizes and ages, on Earth and other planets.

Some of these data, for what are thought to be actual meteorite impact craters are tabulated here. Crater diameter is reported in meters (m) and KE in joules (J).

Crater	Diameter(m)	Kinetic Energy of Impact (J) x 10^{18}
Brent	3800	2.461
Deep Bay	12000	15.85
Boltysh	23000	310
Clearwater Lake West	32000	1000
Manicouagan	70000	14500
Sudbury	140000	205000

Source: Roddy: Dence et al., 1977.

We can use mathematical regression on these data to **derive an equation** relating these two variables:

Notation

KE *= kinetic energy (in J) released by the impact of the meteorite*

D *= diameter (in meters) of the resulting crater*

This will enable us to calculate KE of impact for any Earth crater of known diameter, such as Meteor Crater. However, our ultimate target (so to speak!) is the *volume* of the meteorite, and for that we will need two more equations:

Information:

This equation models ***kinetic energy,*** $KE = 0.5Ms^2$

where *M* **=** ***mass of the meteorite in kilograms (kg)*** ***s=*** ***velocity of the meteorite in meters per second (meters/sec)***

How fast do meteorites typically travel? **The average is about** *20,000 meters/sec***,** so let's use that value for s.

Information

This equation models ***density,*** $\rho = \frac{M}{V}$

ρ **=** ***density of the meteorite in kilograms per cubic meter (kg/meters3)***

V **=** ***volume of the meteorite in cubic meters (meters3)***

Solve the Problem:

Use the online applets or your calculators to answer the questions below.

Problem 1: Make points from the data provided in the table: the first coordinate should be the diameter of the crater and the second coordinate should be the kinetic energy. Write the second coordinate as shown in the table, remember that the real value for KE must be multiplied by 10^{18}. Use the power regression applet or calculator.

Problem 2: Use power regression to find the equation of best fit.
(Round constants to 3 decimal places.)

Answer: $KE = 2.499D^{3.250} \cdot 10^{6}$

Information and Notation:

The equation as given by the power regression applet has the general form

$$y = ax^b$$

In this problem

- $y \cdot 10^{18}$ represents KE, the kinetic energy in Joules (J), and
- x represents ***D***, the diameter of the crater in meters

So the form is $y \cdot 10^{18} = aD^b \cdot 10^{18}$. '$a$' and '$b$' are constants determined by the regression of the data. The coefficient 'a' should be in scientific notation. In this case,

- $a = c \cdot 10^{-12}$ with ***c*** in the form x.xxx).

We can express this equation as: $KE = y \cdot 10^{18} = (aD^b)10^{18} = (c \cdot 10^{-12})D^b \cdot 10^{18} = cD^b \cdot 10^6$

We will use this equation for craters with diameters between 500 and 140,000 meters, 500 < D < 140,000.

Information: Meteor Crater is about 1,200 meters in diameter.

Problem 3: Now you are ready to determine the size of the meteorite that formed Meteor Crater.

a. Use the regression equation you developed to find the kinetic energy (KE) of impact. You can do this calculation in the Math Pad in the Math Tool Chest. [Use Math Pad]

   Answer: $KE = 2.499(1200)^{3.250} \cdot 10^6$ Joules

b. Use the kinetic-energy equation to find the mass of the meteorite. As noted above, use s = 20,000 meters/sec. Record your answer. *[Use Math Pad* or Calculator*]*

   Answer: *$m = 1.271 \times 10^8$ kg*

c. Use the density equation to find the volume of the meteorite. The iron-nickel fragments found at the site have a density of about 7,800 kg/meters^3. Record your answer. *[Use Math Pad]*

   Answer: V = 16295 cubic meters

d. Compare the size of the meteorite to the size of something else familiar to you.

Assume The meteorite was approximately spherical as it plunged to Earth.

Information You can find its radius using the equation for the volume of a sphere,

$V = \frac{4}{3}\pi r^3$ *where* ***V*** = *volume of the meteorite in cubic meters (meters^3)*
r = *radius of the meteorite in meters*
$\pi \approx 3.14159$, ***In Math Pad type 'pi' for π***

Follow-up Questions

1. Review the concepts from geology that were used in this study.
2. Review the math you used to answer the questions above.
3. Do you think the equations you used give accurate estimates of the size of the meteor?
4. How do scientists find equations like this?

Level 0 Age of the Universe

0-1

Directions: Worksheets follow Kéyah Math Module (Kéyah Math)
Answer each of the following questions, showing all work: graphs, tables and calculations.
You will need graph paper for the problem set.

Questions for thought:

1. What do you think of when you think of the universe?

2. What do you know about the age of the universe?

3. How do you think scientists might obtain an estimate of the age of the universe?

Notation:

Basic units

- Because astronomical distances are so large, astronomers like to use a large measure, a parsec. One parsec (pc) is approximately 3.26 light years, or 3.1×10^{13} kilometers (km). (One light year is the distance that light travels in one year.)
- In this study, the distance d of a galaxy from Earth is given in Megaparsecs (Mpc)
- Mega means million, so 1 Mpc = 1×10^6 pc.
- 1 Mpc = 3.1×10^{19} km

Assumption
The relation between distance and velocity is linear.

$d(x10^3)$	$v(x10^3)$
0.028	2.7
0.076	4.2
0.108	10.5
0.136	14.1
0.153	10.5
0.226	13.2
0.283	19.8
0.359	28.2
0.363	20.7
0.408	29.4
0.438	31.8
0.472	44.4
0.476	32.1
0.476	37.2
0.493	33
0.556	34.5
0.639	46.5

Table 0-1

Constructing the Hubble Diagram

The data points to the left are taken from a plot of redshift z of receding galaxies against their distance d from Earth (Galaxies and Cosmology, Jones and Lambourne, page 243) with the second row indicating

redshift x velocity of light = velocity of recession of galaxy = v

The numbers in the distance column give distance d of the galaxy from Earth in Megaparsecs x 10^3 (Mpc); the velocity is kilometer per sec x 10^3. So, for example, a galaxy which is 0.028×10^3 Mpc from Earth is receding at a velocity of 2.7×10^3 km/sec.

Linear Models

In this part, you will find a linear function to express the relationship between distance and velocity. Specifically you will:

* express velocity as a function of distance from the earth,
* find the Hubble constant, and
* determine how fast the Whirlpool Galaxy is receding from us.

Problem Set *(Round off to three decimal places for this work.)*

Question 1: Plot the points corresponding to the data in Table 0-1. The first coordinate is distance from the earth; denote this by d (units are 10^3 Mpc). The second coordinate is velocity of recession of galaxy, units 10^3 km/sec. Use graph paper or your calculator to plot the points.

Question 2: Now find the line of best fit. (The resulting graph is called a **Hubble Diagram**.)

Question 3: What is the slope of this line? What are the units?

Question 4: The Whirlpool Galaxy is approximately 30 x 10^6 light years away from us. (One light year is the distance that light travels in one year.) How fast is it receding from us?

Information:

We can use the Hubble constant to approximate the age of the universe. The modified equation from #1 (ignoring the very small constant, assuming that when d = 0, then v = 0, and rounding the Hubble Constant to one decimal place),

v = 70.3d, is known as **Hubble's Law**.

Question 5: A Black Hole, Perseus A, has been detected traveling away from us at the rate of 1631 km/s. How far away is it?

Question 6: Recall the well-known formula for distance, **distance = (rate of speed) x (time traveled)**, or **d = vt,** and **substitute this into the Hubble Law for d.** Simplify the result and **solve for time, t**.

Question 7: Using this value for the Hubble constant (replace 70.3) and the answer to Question 6, compute t. This answer is a very good approximation to the current age of the universe.

Kéyah Math Worksheets
Module KM#1
Level 1 Streamflow in the Animas River

Directions: Worksheets follow Kéyah Math Modules (Kéyah Math)
Answer each of the following questions, showing all work: graphs, tables and calculations. ***1-1***

Introduction

This is a study about finding the average amount of water that flows through the Animas River each month. The river has its headwaters above Silverton, Colorado, and flows south to Farmington, New Mexico, where it empties into the San Juan River. This study is concerned with the Upper Animas from its headwaters to Durango. It illustrates (in a simplified way) how scientists can use historic data to predict how much water runs through a stream in a certain time period. This is called *streamflow*, it can be recorded using various units, for example, in cubic feet per second (cfs), or cubic meters per second or cubic feet per month, etc. Basically, it is measured by volume per time increment. If you measure streamflow at some point on a stream as 500 cfs, this means that 500 cubic feet of water passes through that point every second. For example, the streamflow for the Mississippi River at Baton Rouge is 211,000 cfs whereas the streamflow for the Colorado River below the Laguna Dam (on the Arizona/ California border) is 398 cfs. Geologists are interested in streamflow because it affects amounts of sediment carried by streams (this is somewhat dependent on streamflow) and how this might change stream beds and land formation.

Warm-up Questions

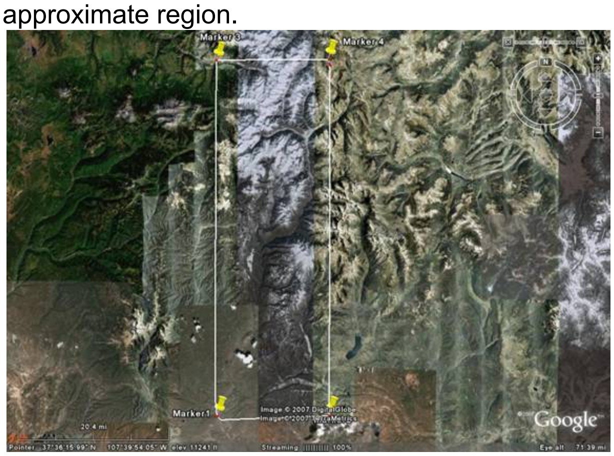
- How much water do you think flows down a stream near you every day?
- How could you measure actual streamflow for this stream?
- Why is it important to predict streamflow for your river or stream?

Using Math to Find the Streamflow for Animas River

The questions that follow will lead you to figuring out the average monthly streamflow for the Animas River at Durango. The first step is to estimate the area of the drainage basin. There are two versions of this module: the first version provides a map of the drainage basin and its approximate area; the second version uses Google Earth and asks you to come up with you own estimate of the area. Either version is available on the website.

Using Math to Find the Streamflow for Animas River (Version 1)

The questions that follow will lead you to figuring out the average monthly stream flow for the Animas River at Durango. We will measure this in cubic feet per month, then convert to cubic feet per second, the most common units used for stream flow in this country.
The first step is to estimate the area of the drainage basin. Figure 1 shows the approximate drainage basin for the Animas from source to Durango. Of course, the actual watershed is not rectangular but this shows the approximate region.



Information you'll need to answer this question is bulleted below, refer to the figure shown, then answer the question below the figure.

- The area of the drainage basin, or watershed, for the Animas from source to Durango is roughly 700 square miles.

Source (Note: This was obtained by computing the area of the rectangle shown. The USGS web site Water Watch (http://water.usgs.gov/waterwatch/?m=real&r=co) shows the actual watershed is 692 square miles. You can check this out by going to the site and locating the dot for Durango in the southwest portion of the state. More information on the watershed is given in the source listed below):
http://www.nmenv.state.nm.us/swqb/Projects/SanJuan/TMDL1/03.pdf

- The total average annual precipitation, including water from snow, over this region is 22.17 inches.

Source (Note: The figure above was obtained by averaging data from these two sources: Monthly weather averages for Silverton, CO from Weather.com and Monthly Climate Summary from the Western Regional Climate Center

Using math to estimate average stream flow for the Animas River

Question 1: What is the area of the watershed in square feet?

(1 mile = 5,280 feet, so 1 square mile = $(5280 \text{ ft})^2 = 5280^2$ square feet)

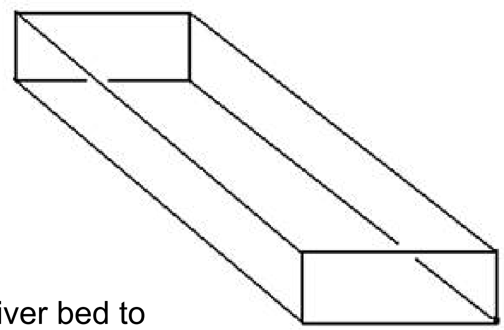
Question 2: Convert the annual amount of precipitation from inches to feet.

(1 foot = 12 inches)

Question 3: Now, find the total volume (in cubic feet—use your answers to #1 and 2) of water from rain and snow that falls on the watershed each year.

Think of the drainage basin as a giant bucket with a rectangular base with area 700 square miles and height the amount of precipitation over the area, 22.17 inches. (See figure below.)

The base of the figure is the rectangular watershed, and the height is the depth of water from precipitation.



Question 4: Only 74% of the rain actually reaches the river bed to contribute to its stream flow (all the rest of the water is evaporated or diverted for other uses). **What is the annual stream flow for the Animas River?**

Question 5: On the average, how much water flows down the river each month? Each second?

(1 year = 365.2 days, 1 day = 24 hours, 1 hour = 60 minutes, and 1 minute = 60 seconds)

Your answer to # 5 is the average stream flow for the Animas River at Durango; the units should be cubic feet per second, or cfs.

Looking back at your answers

- Do you think that the methods used here would be accurate for predicting future stream flow?
- Do you think that you can accurately predict daily, or monthly, stream flow from annual stream flow, particularly for the Animas River? Why?
- What variations in precipitation might affect monthly stream flow?
- How would variations in stream flow affect the stream bed, or the land around the stream?

Go To:
http://pubs.usgs.gov/of/1992/ofr92-129/hcdn92/hcdn/ascii/monthlya/region14/09361500.amm,
find the average of the data given there, and compare to your answers.

Since much of the Animas River watershed lies in the San Juan Mountains at elevations from 8,000 to 14,000 feet, snow and snow melt drastically affects its stream flow. Again, see the website listed above. For a better look at how Animas stream flow is affected by snowfall, go to KM Study #2, "Snowfall and the Animas River Streamflow."

Kéyah Math Worksheets
Module KM#2
Level 2 Volcanic Processes

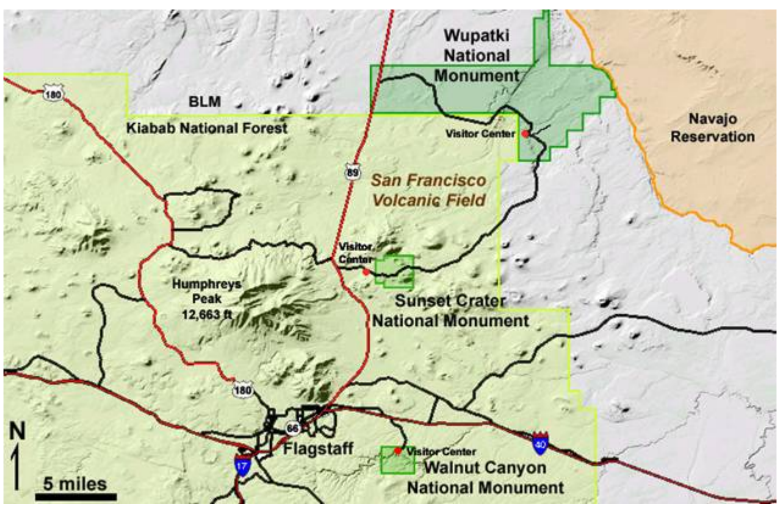
2-1

Directions: Worksheets follow Kéyah Math Modules (Kéyah Math)
Answer each of the following questions, showing all work: graphs, tables and calculations.

Introduction

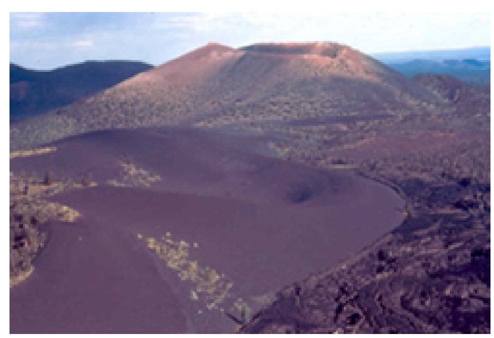
Sunset Crater is located about 15 miles (25 kilometers) northeast of Flagstaff, Arizona (see map below), along the east edge of the San Francisco volcanic field. Volcanic activity in this area was initiated about 6 million years ago and ended about 1000 years ago with the eruption of Sunset Crater (Reynolds et al., 1986; Holm and Moore, 1987; Duffield, 1997. Early eruptions were dominated by the construction of a composite volcano and related domes and flows while later eruptive phases were mostly eruptions of basaltic magmas that formed cinder cones and associated lava flows and spatter cones. More than 600 of these cinder cones have been documented in this region (Duffield, 1997) with Sunset and SP Craters being several of the more well know cones.

Initial eruptions at Sunset Crater ejected volcanic material ranging from fine ash to the size of watermelons (bombs); materials ejected during an eruption are referred to as **tephra**. Erupted bombs associated with this event were formed when sticky blobs of magma were ejected from the vent by gas-charged eruptions. As the hot and sticky masses spun in the air they took on different shapes and forms, some resembling footballs. These eruptions produced a mantle of loose, black material that accumulated around the central vent constructing a nearly symmetrical cone about 1,000 feet (300 meters) high and 1 mile (1.6 kilometers) in base diameter with a 400-foot-deep crater. All of the erupted material associated with this event may have blanketed more than 800 square miles, but presently about 122 square miles are covered with erupted tephra (Holm and Moore, 1987).



A shaded relief computer image of Sunset Crater and other features in the area image from the United States Geological Survey;

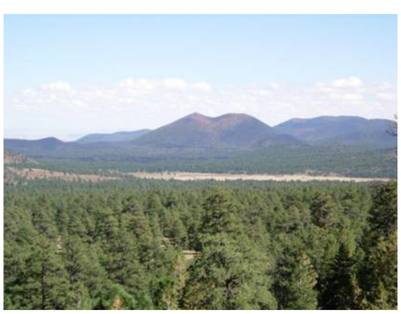
As part of the formation of the Sunset Crater cinder cone, basaltic lava flows (Bonito lava flow and Kana-a flow) erupted along cracks and fissures at the base of Sunset Crater. The Bonito lava flow traveled to the north-northwest for around 3,700 feet (1,130 meters) whereas the Kana-a flow migrated from the east of the cone northeast for about 4 miles (6.4 kilometers). Holm and Moore (1987) also describe small cones to the southeast of the main cinder cone.



Sunset Crater as viewed to the south.

Note the lava flows and other cinder cones in the field

(http://www.nps.gov/sucr/naturescience/volcanoes.htm).



View of Sunset Crater from the west near Sugarloaf Mountain. Photo was taken by David Gonzales

Warm-up Questions

1. How fast do you think a bomb ejected from the Sunset eruption would travel?

2. How far?

3. Would it travel as fast as a Ferrari? A bullet? A shooting star?

4. How much damage would it do to your car if an ejected bomb hit it?

Using math to find the velocity of bombs ejected from the Sunset eruption

Sunset Crater cone dimensions: The Sunset eruption is unusual because the volume of volcanic products (about 0.7 cubic miles, 3 cubic km) is large for a Strombolian event, the air fall dispersal was large, and the discharge rate for magma was high.

Strombolian eruptions are relatively low-level volcanic_eruptions, named after the Italian volcano named Stromboli, where such eruptions consist of ejection of tephra to altitudes of tens to hundreds of meters. They are small to medium in volume, with sporadic violence.

In the last eruption of Sunset Crater in 1064 and 1065 A.D. (Duffield, 1997), fragments of volcanic rock and erupted magma were ejected into the area surrounding the cone.

Before you begin this exercise, it might be useful to visit the Visual Exercises and Component using Stromboli program at: http://www.swisseduc.ch/stromboli/volcano/simulation/index-en.html

The goal of this study is to estimate the initial and terminal velocity of large volcanic bombs ejected from Sunset Crater.

Information and Assumptions

- Sunset Crater cone dimensions: the cone is about 1,000 feet (300 m) high and 1 mile (1.6 km) in base diameter
- Angle of ejection: α = 45 degrees
- The maximum distance that large volcanic bombs travel is 800 meters.

Notation:

- v_0 = initial velocity
- z_0 = height of the volcano
- t_{max} = the maximum time a bomb travels
- u = horizontal distance a bomb travels at time t (see formula 2 below.

The formulas below are only valid when the angle of ejection α = 45 degrees

Formula 1

The height (in meters) at time t is $z = z_0 + \frac{v_0}{\sqrt{2}}t - \frac{1}{2}gt^2$ and $g = 9.8 m/s^2$.

Formula 2

The horizontal distance at time t is $u = \frac{v_0}{\sqrt{2}}t$.

Formula 3

The velocity at time *t* is $V = \sqrt{v_x^2 + v_y^2}$, where $v_x = \frac{v_0}{\sqrt{2}}$ and $v_y = \frac{v_0}{\sqrt{2}} - gt$.

Problem: To find t_{max} , the maximum time a bomb travels, v_0, the initial velocity of the bomb, and V, the total velocity at impact

Question 1: What is the initial velocity? To answer this, follow the steps below. Use meters for height and distance.

A) What is the height and the distance when the volcanic bombs hit the ground? (These are the values of z and u when $t = t_{max}$)

B) Re-write formula 1 for z by substituting z_0= 300 and $g = 9.8 m/s^2$.

C) Use formula 2 and your answer to step A to write an equation for t_{max} in terms of v_0.

D) Substitute your answers from steps A, B, and C in the equation for z when $t = t_{max}$ and solve for v_0.

Question 2: What is t_{max}? (Use formula 2.)

Question 3: What is V, the total velocity at impact? (Use formula 3.)

Question 4: Steve builds a summer home just outside of Flagstaff, but only 2000 meters from Sunset Crater. Suppose there is another eruption from the crater. What will the initial velocity of a bomb need to be to destroy Steve's new vacation home?

Follow-up Questions

5. Describe the concepts from geology that were used in this study.

6. List the math you used to answer the questions above.

7. Do the equations you used give accurate estimates of speed and distance?

8. How do scientists find equations like this?

Kéyah Math Worksheets
Module KM#3

Level 2 Snowmelt and Streamflow of the Animas River

Directions: Worksheets follow Kéyah Math Modules (Kéyah Math)
Answer each of the following questions, showing all work: graphs, tables and calculations. ***3-1***

Introduction

This is a study about finding the average amount of water that flows through the Animas River during the months of snowmelt. The river has its headwaters above Silverton, Colorado, and flows south to Farmington, New Mexico, where it empties into the San Juan River. This study is concerned with the Upper Animas from its headwaters to Durango. It illustrates (in a simplified way) how scientists can use historic data to predict how snow melt affects the amount of water that runs through a stream in a certain time period. The volume of water that flows through a stream at a designated point in a specified time period is called *streamflow*, it can be recorded using various units, for example, in cubic feet per second (cfs), or cubic meters per second or cubic feet per month, etc.

Basically, it is measured by volume per time increment. If you measure streamflow at some point on a stream as 500 cfs, this means that 500 cubic feet of water passes through that point every second. For example, the streamflow for the Mississippi River at Baton Rouge is 211,000 cfs whereas the streamflow for the Colorado River below the Laguna Dam (on the Arizona/California border) is 398 cfs. In regions where there is a lot of snow, streamflow is influenced in a major way during the time when snow is melting. Indeed, the water available for drinking, household use, or irrigation is dependent on the amount of snowfall the region receives in winter.

This is the case for Animas River region since its headwaters and a large part of its path to the San Juan is located in the high San Juan Mountains with elevations from 7,000 to over 14,000 feet. If you completed KM Study #1, "Estimating Average Stream Flow for the Animas River," you found the average streamflow for a year; however the streamflow significantly fluctuates during the year, particularly during early summer when the mountain snow melts.

Geologists are interested in streamflow because it affects amounts of sediment carried by streams (this is somewhat dependent on streamflow) and how this might change stream beds and land formation.

Warm-up Questions

1. What is the streamflow for a river or stream near you?
2. Does the streamflow vary from time to time?
3. Is this stream fed by snow melt at any time during the year?
4. If so, what time of year and what happens to the stream?
5. Is global warming affecting the amount of rain or snow you get each year?

Using Math to See How Streamflow is Affected by Snow Melt (Version 1)

The questions that follow will lead you to predicting the monthly streamflow for the Animas River at Durango during the period when the winter snow is melting. We will measure this in cubic feet per month, then convert to cubic feet per second, the most common units used for streamflow in this country.

Below, Table 1 gives average monthly temperatures, in degrees Fahrenheit, for the drainage basin for the Animas River. This was obtained by averaging temperatures at Silverton and Durango.

Table 1

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Temp	20	24	31	38	47	55	61	60	53	43	30	21

Question 1: Assume that there is no snow melt during the months when average temperature is freezing or below; during this period, snow accumulates.
From the data above, find for which months snow is accumulating.

The data in the Table 2 below gives the average monthly precipitation, including water from snow, in inches, over the Animas River watershed. This was obtained by averaging precipitation at Silverton and Durango.

Table 2

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Precip	1.72	1.73	2.13	1.63	1.58	1.14	2.44	2.80	2.34	2.20	1.74	1.92

Sources: Monthly weather averages for Silverton, CO from Weather.com and Monthly Climate Summary from the Western Regional Climate Center

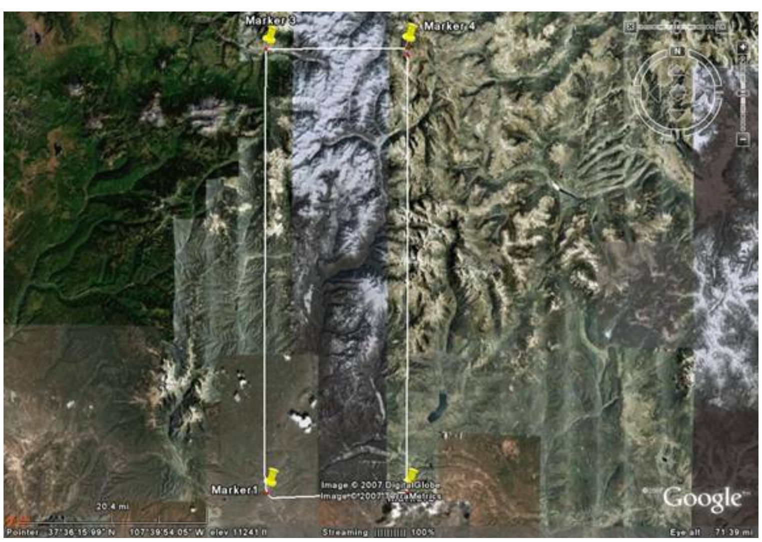
Question 2: From the data above, compute the total water (in inches) from snow during the months of freezing temperatures.

Question 3: Convert your answer to #2 to feet (1 ft = 12 inches).

Question 4: The area of the drainage basin, or watershed, for the Animas from source to Durango is roughly 700 square miles.* **What is the area of the watershed in square feet?**
(Using scientific notation, round to 2 decimal places.)
(1 mile = 5,280 feet, so 1 square mile = $(5280 \text{ ft})^2 = 5280^2$ square feet)

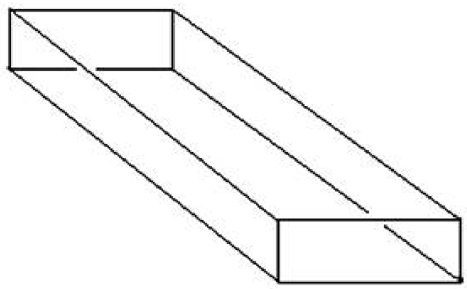
(Note: if you completed KM Study #1, you already have the answer to this question.)

The picture below shows the approximate drainage basin for the Animas from source to Durango. Of course, the actual watershed is not rectangular but this shows the approximate region.



Source (The USGS web site Water Watch) shows the actual watershed is 692 square miles. You can check this out by going to the site and locating the dot for Durango in the southwest portion of the state.)

Question 5: Think of the drainage basin as a giant box with a rectangular base with area 700 square miles and height the amount of water from snow over the area (this is you answer to #3). (See figure left.)



Now, find the total volume (in cubic feet—use your answers to #3 and 4) of water from snow that falls on the watershed during the freezing months.
(Volume = Area of Base x Height; Using scientific notation, round to 2 decimal places.)

Question 6: Assume that it takes 4 months for all the snow to melt after temperatures go above freezing. So, the Animas is getting water from both rain and snow melt during this period. From Table 2, convert the rainfall data to feet for each of the four months of snow melt. (Round to 2 decimal places.)

Question 7: Use your four answers to #6 and your answer to #4 to compute the volume of rain over the watershed for each of the four months of snow melt. Your four answers will be in cubic feet. (Round to 2 decimal places.)

Question 8: As temperatures get warmer in the summer months, snow melts faster. Table 3 below gives the percentage of total snow that melts during each of the four months.

Now, refer to your answer to #5, and find the volume of snow that melts in each of the four months. Your four answers will be in cubic feet. (Round to 2 decimal places.)

Table 3

Month	1	2	3	4
%	2	38	56	4

Question 9: Next, find the total volume of water, rain from #7 + snow melt from #8, over the watershed for each of the four months. (Round to 2 decimal places.)

Question 10: Assume that only 74% of all water actually reaches the river bed (the rest is evaporated or drawn off for agricultural or domestic use). Find the streamflow for the Animas River for each of the four months of snow melt. (Round to 2 decimal places.)

Question 11: .Convert your answers to #10 to cubic feet per second. (Round to nearest cubic foot.)

(1 day = 24 hours, 1 hour = 60 minutes, and 1 minute = 60 seconds)

Looking back at your answers

1. Review the method you used to find the streamflow.
2. Review the math you used to solve the problems above.
3. Do you think this is an accurate way to predict streamflow during the time of snow melt?
4. How are stream beds and erosion affected during times of high streamflow?
5. How would global warming affect streams, land formation, etc.?
6. Compare your answers to the data presented on the website, **http://pubs.usgs.gov/of/1992/ofr92-129/hcdn92/hcdn/ascii/monthlya/region14/09361500.amm**.

Kéyah Math Worksheets
Module KM#4
Level 2 Size of the Earth

Directions: Worksheets follow Kéyah Math Modules (Kéyah Math)
Answer each of the following questions, showing all work: graphs, tables and calculations.

4-1

Warm up question:

Without leaving this country, how could you figure out how far it is all the way around the World?

Introduction

Around 250 BC, at noon on the day of the summer solstice (when the sun is at its highest point in the Northern Hemisphere) in Syrene, Egypt, sunlight filled the vertical shaft of a well; this indicates that the sun is directly overhead, so a vertical pole would cast no shadow. Eratosthenes, who lived in Alexandria, heard of this from a traveler. So on the same day, different year, he noticed that in Alexandria, some 800 kilometers (km) away, a vertical pole cast a shadow. From these observations, he made two deductions:

A. the earth is curved;

B. found the first estimate for the circumference of the Earth.

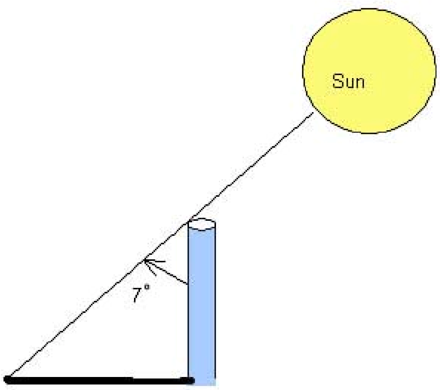


Figure 1

The Earth is Spherical

He measured the angle made by the pole and a line joining the tip of the shadow and the top of the pole (see Figure 1) and found the angle to be about 7^{o}. Then he assumed that light rays from the sun to the Earth were essentially parallel since the sun was so far away and the Earth was so small relative to the sun. From this, and his observations in Alexandria and Syrene, he concluded that the Earth must be curved (see Figure 2), and therefore must be spherical.

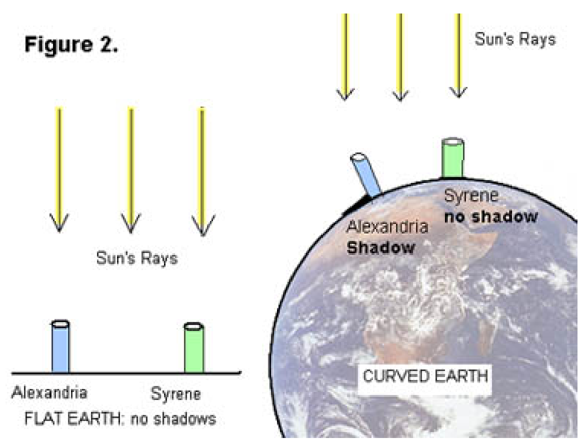


Figure 2.

Using Math to Find the Circumference of the Earth

Next, he used all this information to obtain the first nearly accurate estimation of the circumference of the Earth. Here's how: In the (not-to-scale) Figure 3

A denotes the base of the pole in Alexandria;

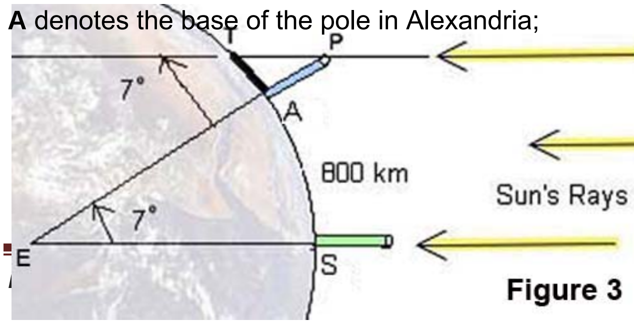


Figure 3

S the base of a pole in Syrene;
T the tip of the shadow cast by the pole in Alexandria;
P the top of the same pole;
E the center of the Earth.

Angle **APT** was measured to be 7°, so by Euclidean geometry interior angles $\angle APT$ and $\angle AES$ are equal, thus angle $\angle AE = 7^\circ$.

There are 360° in a complete circle, so the portion of the circumference of the Earth between **A** and **S** is

$\frac{7^\circ}{360^\circ}$, which is approximately $\frac{1}{50}$ (or, $\frac{360^\circ}{7^\circ}$ is approximately 50).

The distance from Alexandria to Syrene is 800 km, so he concluded that the circumference of the Earth must be $50 \times 800 = 40{,}000\ km$!

This estimate is very close to modern accurate measurements, so Eratosthenes gets credit for the first calculation of the size of the Earth.

We can get a slightly different answer if we compute more accurately:

$\frac{360}{7} = 51.4$, so if we multiply $51.4 \times 800 = 41{,}120\ km$

Questions

Notation:

Some formulas you'll need (r = radius of circle / sphere)

Circumference of a Circle: $C = 2\pi r$ Volume of a Sphere: $V = \frac{4}{3}\pi r^3$

Question 1: What is the radius of the Earth?

Use Eratosthenes' estimate for the circumference of the Earth to find its radius.
(Round your answer to 1 decimal place.)

Question 2: What is the volume of the Earth?

Use your answer to Question 1 to compute the volume of the Earth. (Round your answer to 3 decimal places.)

Here are some follow-up exercises:

3. Study Eratosthenes' method so you completely understand it.
4. There is a follow-up module to this one: it shows how to repeat this activity using locations in Arizona and suggests that you write a module using places in New Mexico, or elsewhere.

Kéyah Math Worksheets
Module KM#5
Level 2 Size of the Earth

Directions: Worksheets follow Kéyah Math Modules (Kéyah Math)

5-1

Answer each of the following questions, showing all work: graphs, tables and calculations.

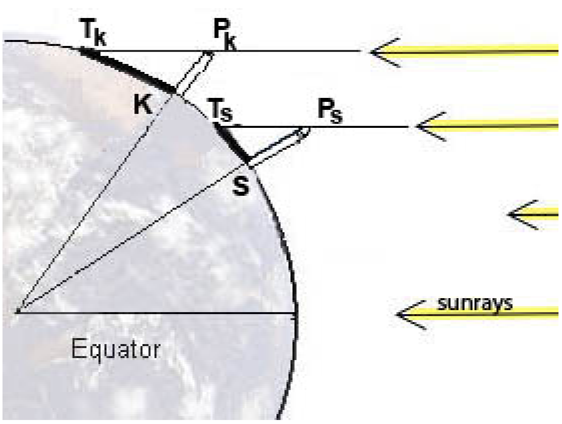
Introduction

This set of exercises gives you the opportunity to use the method of Eratosthenes locally (Arizona) to estimate the circumference of the Earth. Around 250 BC, at noon on the day of the summer solstice (when the sun is at its highest point in the Northern Hemisphere) in Syrene, Egypt, sunlight filled the vertical shaft of a well; this indicates that the sun is directly overhead, so a vertical pole would cast no shadow. Eratosthenes, who lived in Alexandria, heard of this from a traveler. So on the same day, different year, he noticed that in Alexandria, some 800 kilometers (km) away, a vertical pole cast a shadow. From these observations, he deduced that the earth is curved and he estimated the circumference: Module KM#4 provides a detailed description of this. (You can refer to that module for hints.)

This module is slightly different in that the locations we use are farther north than the ones Eratosthenes used, so the angles are different. We suppose the measurements here are taken at noon on the fall equinox, this is when the sun is directly above the equator on its path from summer to winter in the Northern Hemisphere. We use stations just south of Kaibito, Arizona and in Superior,Arizona (instead of Syrene and Alexandria, in Egypt). Kaibito is a small village on the Navajo Reservation in northern Arizona, almost due north of Superior, this will be one of the stations we use to complete these calculations. The other station is Superior, Arizona, a copper mining town about 60 miles east of Phoenix.

Using Math to Estimate the Circumference, Radius and Volume of the Earth

Vertical poles are erected at both stations and angles are measured similar to the way they were measured in by Eratosthenes. In the (not to scale) Figure 1:



S = the base of the pole in Superior, AZ;

K = the base of the pole in Kaibito, AZ;

$\mathbf{T_S}$ = the tip of the shadow cast by the pole in Superior;

$\mathbf{T_K}$ = the tip of the pole's shadow in Kaibito;

$\mathbf{P_S}$ = the top of the pole in Superior;

$\mathbf{P_K}$ = the top of the pole in Kaibito;

E = the center of the Earth

The measurements give these values:

$\angle SP_ST_S = 33.3°$ (try to measure this yourself on the next equinox);

$\angle KP_KT_K = 36.6°$ The direct distance from Superior to Kaibito is 367 km.

Question 1: What is the circumference of the Earth?

Hints: First label the figure with the measurements you have been given.

Can you determine the measure of $\angle KES$? Now follow the method of Eratosthenes to obtain your own estimation of the circumference of the Earth.

Question 2: What is the radius of the Earth?

A. Compute the radius of the Earth using your estimate of the circumference.

(Round your answer to 1 decimal place.) Circumference of a circle: $C = 2\pi r$ where r is the radius.

B. Convert your answer to part A to meters (1 km = 1000 meters).

Question 3: What is the volume of the Earth?

Compute the volume of the Earth using your answer to Exercise 2 B.

(Round your answer to 3 decimal places.)

Volume of a Sphere: $V = \frac{4}{3}\pi r^3$ where r is the radius.

Here are some follow-up exercises:

1. If you know some trigonometry you can use the information to determine the radius and then the circumference. Recall that the arc length $KS = r\theta$ where q is the measure of $\angle KRS$ **in radians.** Convert degrees to radians and the solve for r. Now use this to determine the circumference and volume.

2. Look at a topographical map of New Mexico or Arizona and **choose two locations** to use to measure the circumference of the Earth. You'll need to know the latitudes and longitudes of your chosen locations in order to find the angles, and the places must be on approximately the same longitude (why?). You can also use the internet to find this information, go to www.topozone.com. Don't choose places too close together because this will decrease your accuracy. You'll need to estimate the direct distance between the locations—use the map scale or find it on the internet.

3. There are other variations on this exercise, for example, you can **change the time of year**, maybe to a solstice. You'll need the fact that the Earth's axis is tilted 23.5^o to the plane of the solar system. Or you can **change the location** to your favorite region of the World.

Kéyah Math Project
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Kéyah Math Worksheets
Module KM#6
Level 2 Epicenter of an Earthquake

Directions: Worksheets follow Kéyah Math Modules (Kéyah Math)
Answer each of the following questions, showing all work: graphs, tables and calculations.

6-1

This is a study illustrating how earthquakes are located using data received at seismic stations. In particular, a 3.1 magnitude earthquake is used in this study. You will learn how geologists found out where it occurred.

Warm-up Steps

- Do you know of any recent earthquakes near where you live?
- If so, was any damage done?
- Where have some earthquakes that you know about occurred?
- How do you think that scientist know where an earthquake happened?
- Why is it important to know where an earthquake occurred?

Introduction

An **earthquake** occurs when rocks in the crust move. This movement releases energy that is transmitted outward as **seismic waves**. **Seismic stations** have instruments that can sense and record seismic waves, even those from earthquakes that occur thousands of miles away from the station.

Earthquakes generate several different types of seismic waves, and these waves each travel at different speeds through rock. The fastest waves (traveling at about 6 km/sec through most crustal rocks) are the first to arrive at seismic stations and are called primary (**P**) waves. The next fastest waves travel at about 3.5 km/sec, so they arrive at seismic stations later and have been termed secondary (**S**) waves. The farther these waves travel away from the source of the earthquake, the more the P waves will outpace the S waves.

The **epicenter** of an earthquake is a virtual point on the surface that is located directly above the source of the earthquake. The farther a given seismic station is from an epicenter, the longer the **time interval** between the arrival of the P waves and the arrival of the S waves.

This time interval can be expressed mathematically as a function of the distance from the epicenter and the speeds of the P and S waves through rock. If we know what those two speeds are, and measure the time interval between the P and S arrivals at the seismic station, we can easily calculate how far the epicenter was from that seismic station.

The sequence of problems next will show you how this is done.

6-

Using math to find the epicenter of an earthquake

Notation
The variables are:

t_p = number of seconds a P wave travels after the instant of the earthquake;

t_s = number of seconds a S wave travels after the instant of the earthquake;

d_p = distance (in km) P waves have traveled in t_p seconds; and

d_s = distance (in km) S waves have traveled in t_s seconds

Step 1: Write an equation that shows how far each type of wave traveled after *t* seconds.

Hint: Use the well-known formula, distance = rate x time,

to write the two equations that express distance traveled by the waves in terms of lapsed time. Your answer should be written in the form

$d_p = m_p t_p$, and $d_s = m_s t_s$, where m_p and m_s are the slopes of the lines.

Step 2:

A) Solve each of the two equations from Step 1 for times, t_s and t_p. Since the distance is the same for each wave, denote this by **D** (= $d_p = d_s$).

B) Take the difference of the results from Part A; be sure your answer is positive. Denote this difference by the variable **U**.

C) What does your answer to Part B represent?

D) Solve the equation from Part B for distance **D**.

The table below shows four seismic stations near New Mexico (represented by 3- and 4-letter codes) with latitude and longitude coordinates given in columns 2 and 3. Columns 4 and 5 indicate the times that P and S waves were received at the stations. The last three blank columns are for arrival time of P- and S-waves differences **U**, distances **D** from the stations to the epicenter of the earthquake, and scaled down distances for the map shown below.

Station	**Arrival Time P-waves**	**Arrival Time S-waves**	**Time Diff U**	**Dist to Epicenter D**	**Scaled Distance for Map**
1. TUC	**3:31:25**	**3:31:54**			
2. ANMO	**3:31:28**	**3:32:00**			
3. GDL2	**3:31:45**	**3:32:29**			
4. LTX	**3:32:22**	**3:33:32**			

6-3

Step 3:

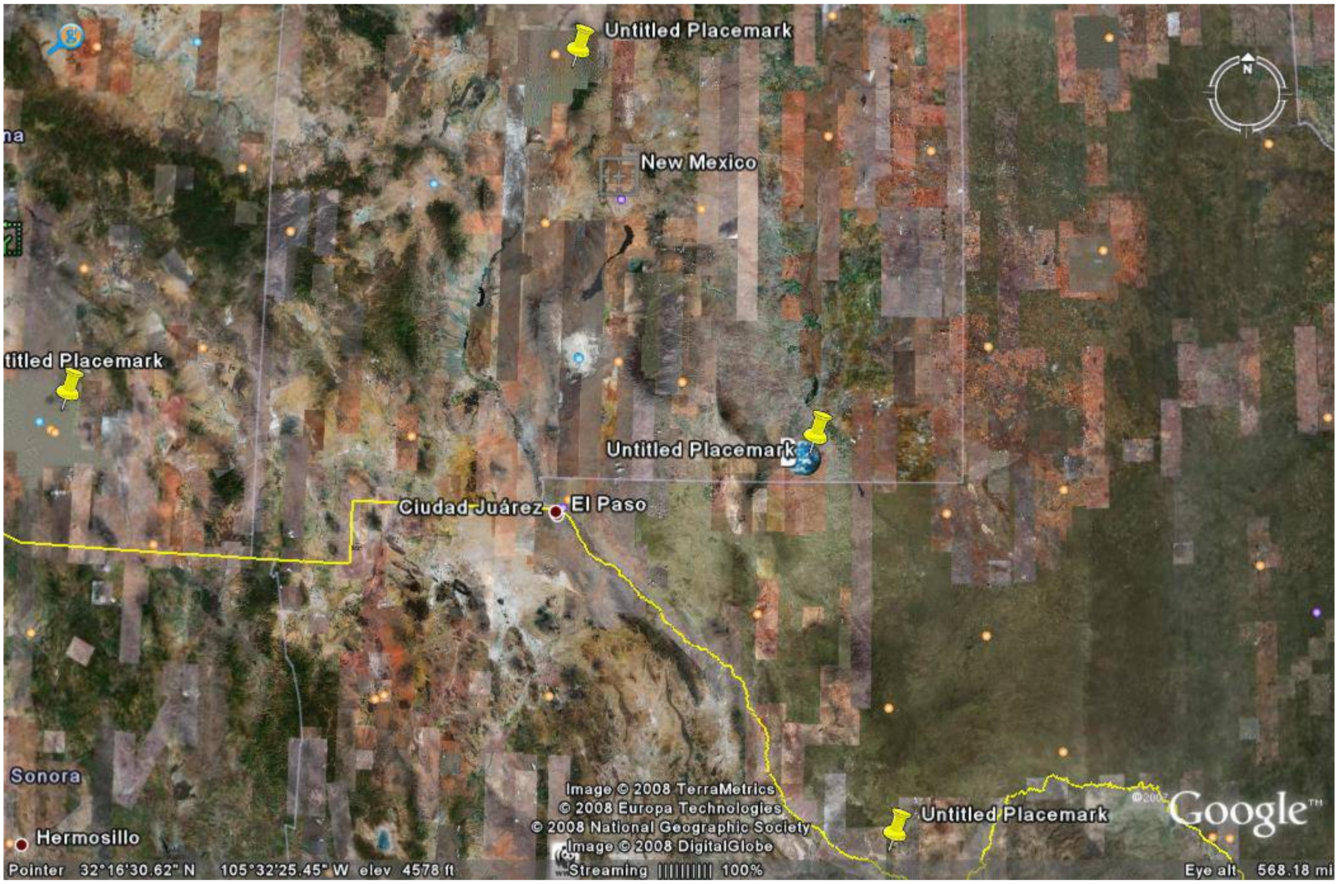
A) From the data given in the table above, compute the time differences in seconds that P and S waves were received at the stations. Complete the column for **U** in the previous table.

B) Use your answers to Part A and the equation from Step 2B to find the distances from the stations to the epicenter of the earthquake to fill in the column for **D**.

The distances you found in Step 3 can be used to find the epicenter of an earthquake. You know how far away the epicenter is from any one station but you don't know what direction. For example, if the epicenter is 1000 km away from the seismic station, you know it must be somewhere on the circle centered at the seismic station with radius 1000 km.

The map on this page is to be used to locate the epicenter. First, print this map.

(You can copy and paste it into a full sheet document, then print for a larger view.)



Step 4.

This a Google Earth map. The four markers locate the four stations listed in the table. From the latitude and longitude coordinates in the table, locate and identify each station.

Step 5.

Next, you will need to figure out the scale on the map.

A) Measure, in inches, the east-west distance on the map along the southern border of New Mexico from Texas to Arizona.

6-4

The real distance between these locations is 565.56 kilometers (Google Earth).

B) Now use this information to figure the scale for this map in km/inch.

Step 6.

This is the final step!

A) Convert the distances **D** to fit the scale on the map; fill in the last column, **Scaled Distance for Map**, in the table with these data.

B) Use the distances you computed in the table for **U** and the scale for the map to draw a circle with center at each station and radius the scaled distance from that station to the epicenter.

C) Look at these circles and locate the epicenter.

D) What are the coordinates of the epicenter? What city or town is it near?

Note: P- and S-wave travel is actually non-linear. For simplicity, we use a linear approximation in this study.

Follow-up Steps

4. Review the geology concepts used in this study.
5. Review the math that you used to find the epicenter.
6. Why is three the minimum number of stations that would be necessary to locate the earthquake?

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Kéyah Math Worksheets
Module KM#7
Level 2 Impact Processes: Meteor Crater

Directions: Worksheets follow Kéyah Math Modules (Kéyah Math)

7-1

Answer each of the following questions, showing all work: graphs, tables and calculations.

Meteor Crater is a distinct impression in the relatively featureless high steppe to the west of Winslow, Arizona. If you were, or are, seeing it for the first time, could you tell how it was formed?

It so happens that Meteor Crater is located in the vicinity of three different volcanic fields in the southwestern Colorado Plateau, so it is perhaps not surprising that some geologists initially thought it was formed by an explosive volcanic eruption. However, no volcanic rocks occur there. Instead, the Crater was littered with small balls of an iron-nickel alloy recognized immediately as having come from a meteoric source. A scientific debate, which extended from the late 19th into the early 20th Century, was finally resolved in favor of an origin brought about by the impact of an iron-nickel *meteorite*.

The largest impacts in Earth's past would have released tremendous energy, and blasted enough material into the upper atmosphere to at least temporarily change climates. Such catastrophes are thought to have triggered or at least contributed to mass extinctions of life on Earth, such as the extinction of the dinosaurs about 65 million years ago. Another impact like that would pose a direct threat to the future of humanity and all other life on Earth, so it is in our best interest to understand the processes and the effects of impact cratering as thoroughly as we can!

An important question that geologists would like to answer is, "What was the size of the meteorite that formed Meteor Crater?" This is the problem that we will answer in this exercise.

Warm-up Experiment (Optional)

Warm-up Questions

5. Have you ever seen a crater that was formed by a meteor impact?

6. After looking at pictures of Meteor Crater, how wide and how deep do you think it is?

7. How big would the meteor have been to form this size crater?

8. How fast would it have been travelling?

Method of Attacking the Problem

Think about factors that would determine how "large" a crater is formed. In part, this would depend on geological conditions specific to the impact site, such as the mechanical properties of soils and rocks. However, one might also assume that kinetic energy, KE, of the meteorite is an important factor: the more energy delivered upon impact, the "bigger" the crater that is excavated.

There is a mathematical relationship between KE and crater diameter, we can measure the diameter of Meteor Crater and calculate the KE, and then the *mass*, of the meteorite. From the mass, we can then calculate the "size" (more precisely the *volume*) of the meteor, because volume and mass are related by *density*, and we have actual fragments of the meteorite on which density has been measured. So, here's an outline of the four-step approach we'll take: ***7-2***

- Use a formula relating KE and diameter of the crater to find KE released on impact
- Use a formula relating KE and mass to find the mass of the meteorite
- Use a formula relating mass density and volume to find the volume of the meteorite
- Use a formula relating volume to radius to find the diameter of the meteorite

Important information you will need for Questions 1 and 5:

The diameter of meteor crater is 1200 meters

Question 1: How much energy was released when the meteorite formed the crater?
Read the information below for help with this question.

We will give you a formula that relates kinetic energy, KE, and diameter D of the crater. (If you want to see how this equation was obtained or find the equation yourself, see Kéyah Math website.)

$$\mathbf{KE = 2.499D^{3.250}x10^{6}}$$

The diameter D of Meteor Crater is measured in meters, and the answer for KE is in Joules. (As a rough guide, 1 joule is the amount of energy required to lift a one kilogram object up to a height of about 10 centimetres on the surface of the Earth by the most efficient method.)
Use this equation to find the kinetic energy released from the meteorite that formed meteor crater.

Question 2: What was the mass of the meteorite?

You can answer this question using two additional pieces of information.

Information

The equation relating mass to kinetic energy: $KE = 0.5Ms^2$

where M is the mass (in kilograms, kg) of the meteorite, and **s** is its velocity (in meters per second).

An estimate for *s*, the velocity of the meteor. How fast do meteors typically travel? The average is about *20,000 meters/sec*, so use that value for s.

Question 3: How big was the meteorite, that is, what was its volume?

Again, you can answer this question by using two more bits of information and your previous answers

Information

The equation relating density, mass, and volume: $\rho = \frac{M}{V}$ where ρ (the greek letter "rho") is the density (in kilograms per cubic meter), M is the mass, and V is the volume (in cubic meters).

Fragments found at the site are iron-nickel, so the meteorite is assumed to be iron-nickel with density $\rho = 7800 kg/meter^3$.

Question 4: What was the diameter of the meteorite?

Notation:

Assuming that the meteorite was spherical, you'll need the formula for the volume V of a sphere:

$V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere (in meters).

Question 5: How many times larger was the diameter of the crater than the diameter of the meteorite?

Follow Up Questions:

5. Review the concepts from geology that were used in this study.
6. Review the math you used to answer the questions above.
7. Do you think the equations you used give accurate estimates of the size of the meteor?

   Why or why not?

8. How do scientists find equations like this?

Kéyah Math Worksheets

Module KM#8

Level 2+ Age of the Earth

Directions: Worksheets follow Kéyah Math Modules (Kéyah Math)

Answer each of the following questions, showing all work: graphs, tables and calculations. ***8-1***

Introduction

How can we tell how old the Earth is?

Certain natural phenomena or processes, such as Earth's year-long solar orbit, and the resulting annual climatic variations that govern the growth of tree rings, can be used as "natural clocks."

If we can find and date a rock that we know has been around since the Earth formed, we can measure the age of the Earth. Can we find in rocks a natural clock that has been operating since they formed? It was discovered that some chemical elements, notably uranium and thorium, are strongly radioactive. These elements occur naturally in nearly all rocks, and they account for the radioactivity you could observe with a Geiger counter.

The radioactive decay process can be described simply as the transformation of an unstable radioactive atom (called the parent) to a new atom (called the daughter) that may differ in atomic number, atomic mass, or both. The transformation occurs either by loss of particles from, or addition of particles to, the parent nucleus.

In some parent-daughter pairs, the daughter is still radioactive and subject to further decay to a new daughter. In other cases, decay yields a daughter that is non-radioactive (stable) and will remain unchanged for the rest of time. The time interval it takes for the parent atoms to decay by half is always the same, no matter how much of the parent element remains. This constant length of time is called the ***half-life***.

How does radioactive decay serve as a "natural clock"?

Some common rocks are weakly radioactive. Numerous chemical analyses of crustal rocks have revealed that radioactive isotopes of elements such as uranium, thorium, potassium, and rubidium occur naturally in these rocks and account for their radioactivity. The precise half-lives of these isotopes have been measured experimentally.

These radioactive isotopes and their half-lives can be used as our natural clock, i.e., we can find out how old certain rocks are from this information.

Part 1. Using Math to Find the Age of Rock in Southwest Colorado

In this section, we will guide you through the process of finding the age of a sample of gabbro found at Electra Lake, just north of Durango in southwest Colorado. Following this, you can repeat these steps to estimate the age of the Earth.

Rubidium-Strontium Dating

Rubidium (^{87}Rb) decays to strontium (^{87}Sr) and because the half-life is so long, it is used by geologists to find the age of very old rock. The isotopes that are used for dating are ^{87}Rb, ^{87}Sr, and ^{86}Sr. ^{87}Rb decays to ^{87}Sr; ^{86}Sr is not a product of decay but is used as a reference isotope. This isotope system can be used as a natural clock to determine the age of many old rocks. This method is called Rubidium-Strontium dating by geologists.

The decay constant λ

Below is a table of the parent-daughter pair (or isotope system) that we will use in radiometric dating of the Electra Lake gabbro and the Earth. The half-life is given in million (10^6) years.

Isotope System		*Half-life T*
Parent isotope (symbol)	*Daughter isotope (symbol)*	*(in million years)*

Rubidium-87 (^{87}Rb)	Strontium-87 (^{87}Sr)	4.88 x 10^4

8-2

The decay constant, usually denoted by Greek letter lambda, λ, is specific to an isotope system. It can be determined experimentally or by using the half-life, and is equivalent to the fraction of atoms that decays per some interval of time. For the Rb-Sr system, $\lambda = 1.42 \times 10^{-5}$.

The Decay Equation

Rocks that contain ^{87}Rb also contain initial amounts of ^{87}Sr, so when comparing the relative amounts of ^{87}Rb and ^{87}Sr, the amount of ^{87}Sr present initially must be accounted for. Also, a certain amount of ^{86}Sr is present that is not a product of Rb decay, this is stable.
So when counting the amount of the daughter ^{87}Sr present now, the amount of ^{87}Sr present initially, $^{87}Sr_{initial}$, must be considered. Also, the amount of stable ^{86}Sr present must be accounted for. At time t = now, the ratio of both $^{87}Sr_{now}$ and $^{87}Rb_{now}$ to ^{86}Sr can be measured; hence the equation that can be used for the dating process is

$$\frac{^{87}Sr_{NOW}}{^{86}Sr} = \frac{^{87}Sr_{initial}}{^{86}Sr} + \frac{^{87}Rb_{NOW}}{^{86}Sr}\left(e^{u} - 1\right)$$

This equation has the form of a linear equation $y = b + mx$, where

$y = \frac{^{87}Sr_{now}}{^{86}Sr}$, $b = \frac{^{87}Sr_{initial}}{^{86}Sr}$, $x = \frac{^{87}Rb_{now}}{^{86}Sr}$, and slope $m = (e^{u} - 1)$.

Now, Rb-Sr dating of a rock incorporates the following procedure that is followed below:

- Rock samples are collected from a site
- The ratios x and y are measured in each sample with a mass spectrometer
- The results are plotted on an (x, y)-axis system (this plot is called an ***isochron diagram***)
- The line of best fit--the line that comes closest to all the points, called the ***regression line***-- for the plot is found using the linear regression applet
- The slope of this line is set equal to m = $(e^{u} - 1)$
- The value of *t* is found from this equation—this is the age of the rock!

(Reference: Looking into the Earth; Musset & Khan; Cambridge University Press 2000)

Information: Five samples of gabbro were collected near Electra Lake in southwest Colorado. Ratios x and y were measured by a mass spectrometer and are listed in the table below.

(Reference: Precambrian Rb-Sr Chronology in the Needle Mountains, Southwestern Colorado; Bickford, Wetherill, Barker, Lee-Hu; J. Geophysical Research, Vol. 74, No. 6, 1969)

X	Y
1.731	**0.7415**
0.580	**0.7155**
0.135	**0.7078**
0.569	**0.7172**
3.198	**0.7700**

The graph shown below indicates the points plotted, the regression line and its equation.

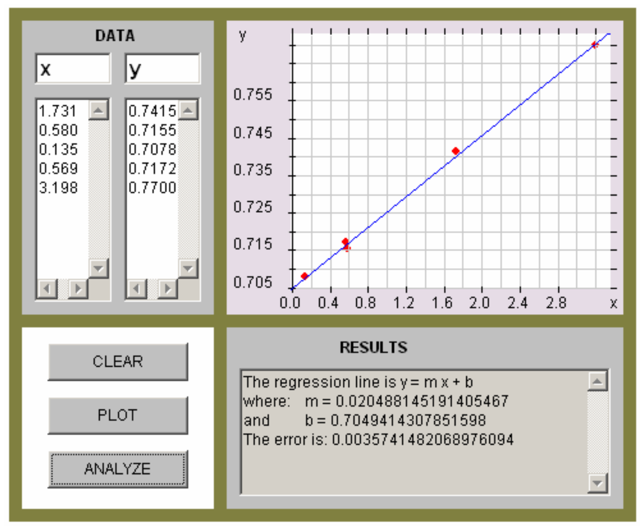
(This graph is from the linear regression applet.)

8-3

The equation of the regression line is

Y = 0.7049 + 0.0205x,

its slope is m = 0.0205.



Question 1: What is the age of the gabbro from Electra Lake?

(Hint: Equate the value for *m* to the slope in the decay equation and solve for *t*. This can be done algebraically using logarithms or by using the Plot-Solve applet.)

For a worked solution to this question, click here.

Answer: The gabbro is, approximately, 1,429 million years old.

Part 2. Using Math to Find the Age of the Earth

Here you will use Rubidium-Strontium decay to date a meteorite samples. Assuming that samples, the Earth, and the entire solar system were formed at approximately the same time, this should give us a good approximation to the age of the Earth.

Meteorites	**87Rb/^{86}Sr**	**87Sr/^{86}Sr**	From five meteorite samples, a mass spectrometer measures the ratios of $^{87}Sr_{now}$ to ^{86}Sr and $^{87}Rb_{now}$ to ^{86}Sr; the data are listed in the table.
Modoc	**0.86**	**0.757**	
Homestead	**0.80**	**0.751**	

Bruderheim	**0.72**	**0.747**	
Kyushu	**0.60**	**0.739**	
Buth Furnace	**0.09**	**0.706**	

(Reference: Rubidium-Strontium Dating of Meteorites; http://hyperphysics.phy-astr.gsu.edu/hbase/nuclear/meteorrbsr.html)

Question 2: How old is the Earth?

In order to answer this question, you can repeat the procedure given in the dating of the gabbro; specifically:

- Plot the data on an (x, y)-axis system to create the isochron diagram
- Use linear regression to find the slope of the line of best fit
- Set the slope equal to $\left(e^{u} - 1\right)$
- Solve for *t*.

(You can solve this either algebraically or graphically—ask your Instructor. You may be provided a print-out of a graph that can be used to find the answer to this question, or you can use a graphing calculator, or go to the Kéyah Math or Earth Math website and use the Plot/Solve applet.)

Kéyah Math Project
An online set of exercises in mathematical geoscience, situated in culturally important places in the Southwest United States.

Kéyah Math Worksheets
Module KM#9
Level 3 Mass & Density of the Earth

Directions: Worksheets follow Kéyah Math Modules (Kéyah Math)

9-1

Answer each of the following questions, showing all work: graphs, tables and calculations.

The goal of this study is to learn how the famous physicist, Sir Isaac Newton, computed the mass of the Earth, and then use this to compute its density.

Basic Concepts:

* Any object in the Universe attracts any other object.

* The force that moves objects toward each other is called ***gravity***.

* The ***mass*** of an object is a fundamental property of the object with measured in ***kilograms (kg)***.

* Even though we often use weight and mass interchangeably in everyday language, the weight of an object depends on the force of gravity. If you went to the moon, your mass would not change but your weight would be much less. The weight of an object is the force of gravity on the object; this can be described by the equation

w = mg where ***m*** is the mass of the object and ***g*** is the acceleration due to gravity.

* The ***density*** of an object is the mass per unit volume.

In the year 1680, Sir Isaac Newton discovered the famous equation known as the Law of Gravitational Attraction on two objects. You will use this result, together with another result also due to Newton, to compute the mass of the Earth.

Notation:

Newton 's Law of Gravitational Attraction, $F = G\frac{m_1 m_2}{r^2}$, describes the force with which two objects attract each other.

- ***G*** is called the universal gravitational constant and its value is known
- m_1 and m_2 denote the masses of the two objects
- ***r*** is the distance between the objects.

For an object with mass m on the surface of the earth, this equation becomes

Equation 1: $F = m\frac{GM_E}{R_E^2}$ where M_E is the mass of the earth and R_E is the radius of the earth.
Newton's Second Law of Motion, *F* = mass x acceleration describes the relationship between force, mass and acceleration. On the surface of the earth the acceleration of gravity is about $g = 9.8\ meters/s^2$ **and we have Equation 2: *F* = *mg***.

Since equations 1 and 2 describe the same force, we have $m\frac{GM_E}{R_E^2} = F_{gravity} = mg$,

which simplifies to our basic equation Basic Equation: $\frac{GM_E}{R_E^2} = g$.

9-2

Thought questions

- If the distance between two objects increases does the force of attraction increase or decrease?
- Which object will have greater acceleration due to gravity, one of very large mass or one of very small mass?

You will use the basic equation $\frac{GM_E}{R_E^2} = g$ to estimate the mass, volume and density of the earth.

Information:

- **The average radius of the Earth is 6.38 x 10**^{6} meters
- The **universal gravitational constant** *G* is $G = 6.672 \times 10^{-8}\ meters/(Mg \cdot s^2)$

where Mg is megagrams (1 Mg = 1000 kilograms) and s is seconds.

- The acceleration of gravity at the surface of the Earth is about $g = 9.8\ meters/s^2$
- The volume of a sphere of radius *r* is given by the formula $V = \frac{4}{3}\pi r^3$
- $\text{Density} = \frac{\text{mass}}{\text{volume}}$

The answers to the following questions will use Newton's methods to determine the mass and density of the Earth.

Question 1: What is the mass of the Earth?

To answer this question, follow these steps:

- Substitute the values given above in the equation $\frac{GM_E}{R_E^2} = g$.

Solve the equation for ME, the mass of the Earth.

Question 2 What is the volume of the Earth?

To answer this question, assume the Earth is a sphere with radius RE. Use the appropriate information given above.

Question 3 What is the density of the Earth?

Use your answers from the first two questions to answer this one.

Here are some follow-up questions to think about:

7. How do you think Newton discovered his Law of Gravitational Attraction?

8. Do you think this is an accurate method for computing the density of the Earth?

9. Do you know of modern, more high-tech techniques that might give a more accurate result?

10. How do you think geologists might measure the density of rocks?

11. Why do you think it's important to know the density of the Earth, or rocks that are part of the Earth?

12. What do you think the Earth is composed of to give it the density that you calculated?

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Kéyah Math Worksheets
Module KM#10
Level 3 Layers of the Earth

10-1

Directions: Worksheets follow Kéyah Math Modules (Kéyah Math)

Answer each of the following questions, showing all work: graphs, tables and calculations.

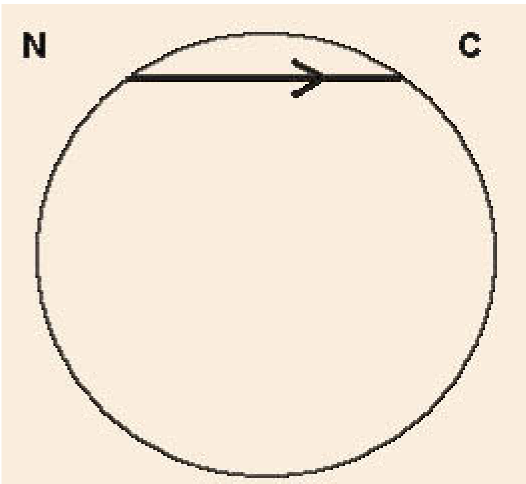

Figure 1

In the following exercises, **refer to Figure 1** (left), and **round all answers to 1 decimal place**.

Information:

You found the radius of the Earth to be 6,490 kilometers using the methods of Eratosthenes. Actually, the earth is not quite spherical, it bulges some at the equator, but the **average radius of the Earth is 6,371 km**, so that's what we'll use for this exploration.

An earthquake occurs at the indicated location labeled N on Figure 1. A seismograph located at a station labeled C which is 80 kilometers from N records the first seismic wave 12.3 seconds after the earthquake occurs.

Question 1: Using the known travel time and distance, compute the velocity of the seismic wave.

ASSUME: Since this is the first wave that reaches station C, we assume that this wave is the one that traveled in a straight line (shortest distance) from N to C. We assume that the distance, 80 km, from N to C along the surface of the Earth is approximately equal to the straight line distance from N to C.

10-2

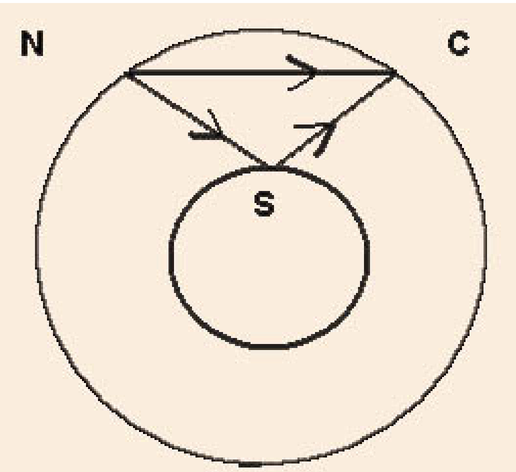

Figure 2

A second wave arrives at station C 18.8 seconds after the earthquake occurs.

Question 2: Find the distance this second wave traveled. (Use the information about the second wave and the velocity you have computed.)

Question 3: Can you guess what path that this second wave traveled?

If you guessed that the wave must have bounced off some barrier inside the Earth, then you are correct! See Figure 2.

Next we will find the depth of this barrier. See Figure 3.

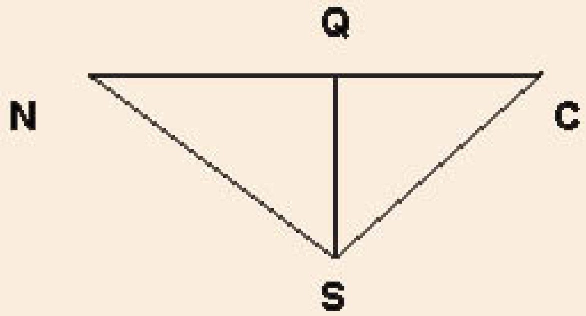

Figure 3

Information:

Laws of physics tell us:

- the **seismic wave bounces off the barrier at the point labeled S at the same angle that it hits the barrier.**

Also, from geometry, we know these facts:

- **the sides NS and SC have the same length;**
- **the line from S to Q divides the triangle NCS into two right triangles NQS and CQS that are the same size.**

Question 4: Compute the depth SQ of the barrier. (Use the information together with your previous answers to compute SQ, the depth of the barrier).

ASSUME: Here again, we assume that the straight line from N to C is the same as the very slightly curved line from N to C, so the length of QS is the same as the very slightly longer distance from S to the surface of the Earth.)

Question 5: Compute the radius of the interior barrier.

ASSUME: The Earth and this newly discovered interior "barrier" are both spherical. This is one way to confirm that the Earth is layered, i.e., there's some inner core that's thick enough to reflect seismic waves!

Kéyah Math Worksheets

Module KM#11

Level 3 Size, Mass & Density of the Earth

Directions: Worksheets follow Kéyah Math Modules (Kéyah Math)

11-1

Answer each of the following questions, showing all work: graphs, tables and calculations.

This study has two parts. In the first part you will estimated the circumference, radius and volume of the Earth. In the second part you will determine the mass and density of the Earth. This module combines the material from KM#4 and KM#9.

Part 1: Estimating the Circumference of the Earth

The goal of this study is to learn how Eratosthenes made the first close estimate of the circumference of the Earth and then use his estimate to compute its radius and volume.

Warm up question:

Without leaving this country, how could you figure out how far it is all the way around the World?

Introduction

Around 250 BC, at noon on the day of the summer solstice (when the sun is at its highest point in the Northern Hemisphere) in Syrene, Egypt, sunlight filled the vertical shaft of a well; this indicates that the sun is directly overhead, so a vertical pole would cast no shadow. Eratosthenes, who lived in Alexandria, heard of this from a traveler. So on the same day, different year, he noticed that in Alexandria, some 800 kilometers (km) away, a vertical pole cast a shadow. From these observations, he made two deductions:

A. the earth is curved;

B. found the first estimate for the circumference of the Earth.

The Earth is Spherical

He measured the angle made by the pole and a line joining the tip of the shadow and the top of the pole (see Figure 1) and found the angle to be about 7^{o}. Then he assumed that light rays from the sun to the Earth were essentially parallel since the sun was so far away and the Earth was so small relative to the sun. From this, and his observations in Alexandria and Syrene, he concluded that the Earth must be curved (see Figure 2), and therefore must be spherical.

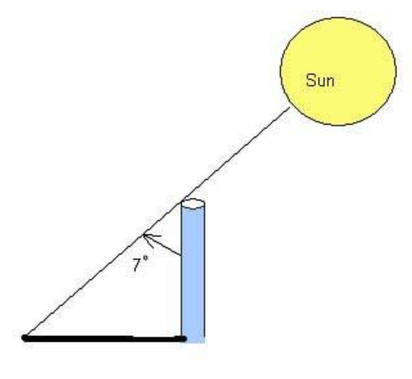

Figure 1

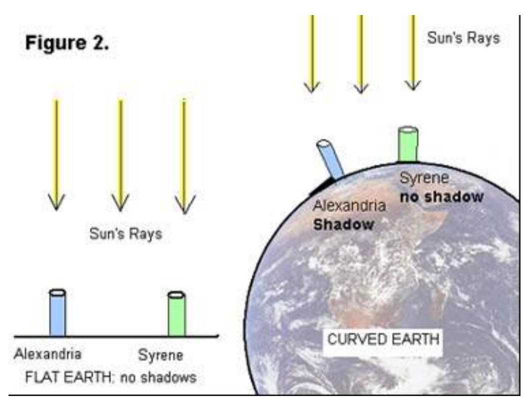

Figure 2.

Using Math to Find the Circumference of the Earth

Next, he used all this information to obtain the first nearly accurate estimation of the circumference of the Earth. Here's how:

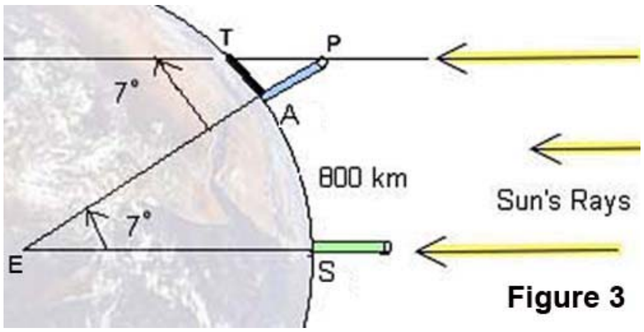

Figure 3

In the (not-to-scale) Figure 3:
A denotes the base of the pole in Alexandria;
S the base of a pole in Syrene;
T the tip of the shadow cast by the pole in Alexandria;
P the top of the same pole;
E the center of the Earth.

Angle **APT** was measured to be 7^o, so by Euclidean geometry interior angles $\angle APT$ and $\angle AES$ are equal, thus angle $\angle AES = 7^o$.

There are 360o in a complete circle, so the portion of the circumference of the Earth between **A** and **S** is $\frac{7^o}{360^o}$, which is approximately $\frac{1}{50}$ (or, $\frac{360^o}{7^o}$ is approximately 50).
The distance from Alexandria to Syrene is 800 km, so he concluded that the circumference of the Earth must be $50 \times 800 = 40{,}000\ km$!

This estimate is very close to modern accurate measurements, so Eratosthenes gets credit for the first calculation of the size of the Earth. We can get a slightly different answer if we compute more accurately: $\frac{360}{7} = 51.4$, so if we multiply $51.4 \times 800 = 41{,}120\ km$

Notation:

Some formulas you'll need (r = radius of circle / sphere)

Circumference of a Circle: **C =** $2\pi r$

Volume of a Sphere: $V = \frac{4}{3}\pi r^3$

Question 1: What is the radius of the Earth?

Use Eratosthenes' estimate for the circumference of the Earth to find its radius. (Round your answer to 1 decimal place.)

Question 2: What is the volume of the Earth?

Use your answer to Question 1 to compute the volume of the Earth. (Round your answer to 3 decimal places.)

Part 2: Determining the Mass and Density of the Earth

The goal of this study is to learn how the famous physicist, Sir Isaac Newton, computed the mass of the Earth, and then use this to compute its density.

Basic Concepts

- Any object in the Universe attracts any other object.
- The force that moves objects toward each other is called ***gravity***.
- The ***mass*** of an object is a fundamental property of the object with measured in ***kilograms (kg)***.
- Even though we often use weight and mass interchangeably in everyday language, the weight of an object depends on the force of gravity. If you went to the moon, your mass would not change but your weight would be much less. The weight of an object is the force of gravity on the object; this can be described by the equation

w = mg where *m* is the mass of the object and *g* is the acceleration due to gravity.

- The *density* of an object is the mass per unit volume.

In the year 1680, Sir Isaac Newton discovered the famous equation known as the Law of Gravitational Attraction on two objects. You will use this result, together with another result also due to Newton, to compute the mass of the Earth.

Notation:

Newton 's Law of Gravitational Attraction, $F = G\frac{m_1 m_2}{r^2}$, describes the force with which two objects attract each other.

- ***G*** is called the universal gravitational constant and its value is known
- m_1 and m_2 denote the masses of the two objects
- ***r*** is the distance between the objects.

For an object with mass *m* on the surface of the earth, this equation becomes

Equation 1:

$F = m\frac{GM_E}{R_E^2}$ where M_E is the mass of the earth and R_E is the radius of the earth.

Newton's Second Law of Motion, *F* = mass x acceleration describes the relationship between force, mass and acceleration. On the surface of the earth the acceleration of gravity is about $g = 9.8\ meters/s^2$ and we have **Equation 2: *F = mg***.

Since equations 1 and 2 describe the same force, we have $m\frac{GM_E}{R_E^2} = F_{gravity} = mg$,which simplifies to our basic equation

Basic Equation: $\frac{GM_E}{R_E^2} = g$.

Thought questions

- If the distance between two objects increases does the force of attraction increase or decrease?

- Which object will have greater acceleration due to gravity, one of very large mass or one of very small mass? *11-4*

You will use the basic equation $\frac{GM_E}{R_E^2} = g$ to estimate the mass, volume and density of the earth.

Information:

- **The average radius of the Earth is 6.38 x 10^6** meters
- The **universal gravitational constant** *G* is $G = 6.672 \times 10^{-8}\ meters/(Mg \cdot s^2)$ where Mg is megagrams (1 Mg = 1000 kilograms) and s is seconds.
- The acceleration of gravity at the surface of the Earth is about $g = 9.8\ meters/s^2$
- The volume of a sphere of radius *r* is given by the formula $V = \frac{4}{3}\pi r^3$
- $\text{Density} = \frac{\text{mass}}{\text{volume}}$

The answers to the following questions will use Newton's methods to determine the mass and density of the Earth.

Question 1: What is the mass of the Earth?

To answer this question, follow these steps:

- Substitute the values given above in the equation $\frac{GM_E}{R_E^2} = g$.
- Solve the equation for ME, the mass of the Earth.

Question 2: What is the volume of the Earth?

To answer this question, assume the Earth is a sphere with radius RE. Use the appropriate information given above.

Question 3: What is the density of the Earth?

Use your answers from the first two questions to answer this one.

Here are some follow-up questions to think about:

1. How do you think Newton discovered his Law of Gravitational Attraction?

2. Do you think this is an accurate method for computing the density of the Earth?

3. Do you know of modern, more high-tech techniques that might give a more accurate result?

4. How do you think geologists might measure the density of rocks?

5. Why do you think it's important to know the density of the Earth, or rocks that are part of the Earth?

6. What do you think the Earth is composed of to give it the density that you calculated?

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Kéyah Math Worksheets
Module KM#12
Level 4 Age of the Earth

Directions: Worksheets follow Kéyah Math Modules (Kéyah Math)

12-1

Answer each of the following questions, showing all work: graphs, tables and calculations.

Introduction
How can we tell how old the Earth is?

Certain natural phenomena or processes, such as Earth's year-long solar orbit, and the resulting annual climatic variations that govern the growth of tree rings, can be used as "natural clocks."If we can find and date a rock that we know has been around since the Earth formed, we can measure the age of the Earth. Can we find in rocks a natural clock that has been operating since they formed? It was discovered that some chemical elements, notably uranium and thorium, are strongly radioactive. These elements occur naturally in nearly all rocks, and they account for the radioactivity you could observe with a Geiger counter.

The radioactive decay process can be described simply as the transformation of an unstable radioactive atom (called the parent) to a new atom (called the daughter) that may differ in atomic number, atomic mass, or both. The transformation occurs either by loss of particles from, or addition of particles to, the parent nucleus.

In some parent-daughter pairs, the daughter is still radioactive and subject to further decay to a new daughter. In other cases, decay yields a daughter that is non-radioactive (stable) and will remain unchanged for the rest of time. The time interval it takes for the parent atoms to decay by half is always the same, no matter how much of the parent element remains. This constant length of time is called the ***half-life***.

How does radioactive decay serve as a "natural clock"?

Some common rocks are weakly radioactive. Numerous chemical analyses of crustal rocks have revealed that radioactive isotopes of elements such as uranium, thorium, potassium, and rubidium occur naturally in these rocks and account for their radioactivity. The precise half-lives of these isotopes have been measured experimentally.

These radioactive isotopes and their half-lives can be used as our natural clock, i.e., we can find out how old certain rocks are from this information.

Part 1. Using Math to Find the Age of Rock in Southwest Colorado

In this section, we will guide you through the process of finding the decay constant for a radioactive isotope in the basic decay equation, and then use this and another decay equation to find the age of a sample of gabbro found at Electra Lake, just north of Durango in southwest Colorado. Following this, you can repeat these steps to estimate the age of the Earth.

Rubidium-Strontium Dating

Rubidium (^{87}Rb) decays to strontium (^{87}Sr) and because the half-life is so long, it is used by geologists to find the age of very old rock. The isotopes that are used for dating are ^{87}Rb, ^{87}Sr, and ^{86}Sr. ^{87}Rb decays to ^{87}Sr; ^{86}Sr is not a product of decay but is used as a reference isotope. This isotope system can be used as a natural clock to determine the age of many old rocks. This method is called Rubidium-Strontium dating by geologists.

The *decay function* is $P(t) = P_0 e^{-\lambda t}$, where

$P(t)$ = number of atoms of the parent isotope at time t,
$P_0 = P(0)$ = initial number of atoms of the parent isotope (at $t = 0$, when decay started),
λ = (Greek letter lambda) the decay constant specific to that parent isotope, which can be determined experimentally, and is equivalent to the fraction of atoms that decays per some interval of time.

We can use the basic decay equation and isotope half-life to find the decay constant for specific isotopes.

Finding the decay constant λ

Below is a table of the parent-daughter pair (or isotope system) that we will use in radiometric dating of the Electra Lake gabbro and the Earth. The half-life is given in million (10^6) years.

Isotope System		Half-life T
Parent isotope (symbol)	Daughter isotope (symbol)	(in million years)
Rubidium-87 (87 Rb)	Strontium-87 (87 Sr)	4.88 x 104

Question 1. What is the value of the decay constant for rubidium-strontium?

Question 2. Suppose that in an isotope system, the parent decays into the daughter, and there was no daughter atoms present initially. What is the ratio of daughter atoms to parent atoms at time *t*?

This solution to Question 2, $\frac{D(t)}{P(t)} = e^{\lambda t} - 1$, shows the desired ratio—***note that this is true for any such isotope system--***this equation is called the ***basic decay equation***.

Modification of the Basic Decay Equation for Rb-Sr Dating

First, solve the basic decay equation for the number of daughter atoms, *D(t)*.

$$D(t) = P(t)\left(e^{\lambda t} - 1\right)$$

Rocks that contain ^{87}Rb also contain initial amounts of ^{87}Sr, so when comparing the relative amounts of ^{87}Rb and ^{87}Sr, the amount of ^{87}Sr present initially must be accounted for. Also, a certain amount of ^{86}Sr is present that is not a product of Rb decay, this is stable.

So when counting the amount of the daughter ^{87}Sr, the basic decay equation must be modified to add in the amount present initially, ^{87}Srinitial, to the amount decayed at time t, ^{87}Srt. This gives the modified equation, $^{87}Sr_t = {}^{87}Sr_{initial} + {}^{87}Rb_t\left(e^{\lambda t} - 1\right)$.

At time t = now, the ratio of both ^{87}Srt and ^{87}Rbt to ^{86}Sr can be measured, so divide both sides of the above equation by ^{86}Sr (which is constant) to get

$$\frac{^{87}Sr_{now}}{^{86}Sr} = \frac{^{87}Sr_{initial}}{^{86}Sr} + \frac{^{87}Rb_{now}}{^{86}Sr}\left(e^{\lambda t} - 1\right).$$

This equation has the form of a linear equation y = b + mx, where

$y = \frac{^{87}Sr_{now}}{^{86}Sr}$, $b = \frac{^{87}Sr_{initial}}{^{86}Sr}$, $x = \frac{^{87}Rb_{now}}{^{86}Sr}$, and slope $m = \left(e^{\lambda t} - 1\right)$.

Now, Rb-Sr dating of a rock incorporates the following procedure that is followed below:

- Rock samples are collected from a site
- The ratios x and y are measured in each sample with a mass spectrometer
- The results are plotted on an (x, y)-axis system (this plot is called an ***isochron diagram***)
- The line of best fit for the plot is found using linear regression
- The slope of this line is set equal to m = $\left(e^{\lambda t} - 1\right)$
- The value of *t* is found from this equation—this is the age of the rock!

(Reference: Looking into the Earth; Musset & Khan; Cambridge University Press 2000)

Information: Five samples of gabbro were collected near Electra Lake in southwest Colorado. Ratios x and y were measured by a mass spectrometer and are listed in the table below.

(Reference: Precambrian Rb-Sr Chronology in the Needle Mountains, Southwestern Colorado; Bickford, Wetherill, Barker, Lee-Hu; J. Geophysical Research, Vol. 74, No. 6, 1969)

X	Y
1.731	**0.7415**
0.580	**0.7155**
0.135	**0.7078**
0.569	**0.7172**
3.198	**0.7700**

The equation of the regression line is Y =

its slope is m = 0.0205.

Question 3. What is the age of the gabbro from Electra Lake?

Part 2. Using Math to Find the Age of the Earth

Here you will use Rubidium-Strontium decay to date a meteorite samples. Assuming that samples, the Earth, and the entire solar system were formed at approximately the same time, this should give us a good approximation to the age of the Earth.

Meteorites	$^{87}Rb/^{86}Sr$	$^{87}Sr/^{86}Sr$
Modoc	**0.86**	**0.757**
Homestead	**0.80**	**0.751**
Bruderheim	**0.72**	**0.747**
Kyushu	**0.60**	**0.739**
Buth Furnace	**0.09**	**0.706**

From five meteorite samples, a mass spectrometer measures the ratios of $^{87}Sr_{now}$ to ^{86}Sr and $^{87}Rb_{now}$ to ^{86}Sr; the data are listed in the table to the left.

(Reference: Rubidium-Strontium Dating of Meteorites; http://hyperphysics.phy-astr.gsu.edu/hbase/nuclear/meteorrbsr.html)

Question. How old is the Earth?

In order to answer this question, you can repeat the procedure given in the dating of the gabbro; specifically:

- Plot the data on an (x, y)-axis system to create the isochron diagram
- Use linear regression to find the slope of the line of best fit
- Set the slope equal to $(e^{\lambda t} - 1)$
- Solve for *t*.

(You can solve this either algebraically or graphically—ask your Instructor. You may be provided a print-out of a graph that can be used to find the answer to this question, or you can use a graphing calculator, or go to the Kéyah Math or Earth Math website and use the Plot/Solve applet.)

Kéyah Math Project
An online set of exercises in mathematical geoscience, situated in culturally important places in the Southwest United States.

Kéyah Math Worksheets

Module KM#13

Level 4 Impact Processes: Meteor Crater

Directions: Worksheets follow Kéyah Math Modules (Kéyah Math)

13-1

Answer each of the following questions, showing all work: graphs, tables and calculations.

What was the size of the meteorite that formed Meteor Crater?

Warm-up Questions

5. Have you ever seen a crater that was formed by a meteor impact?

6. After looking at pictures of Meteor Crater, how wide and how deep do you think it is?

7. How big would the meteor have been to form this size crater?

8. How fast would it have been travelling?

Understand the Problem:

A meteor falling toward the Earth is propelled by gravitational attraction. Because it is moving, the meteorite has an energy of movement or *kinetic energy (KE)*, which is described by the equation:

$KE = 0.5Ms^2$ **where M** is the mass of the meteorite and **s** is its velocity.

If the meteorite is accelerating downward, its KE must be increasing as the *square* of the velocity! If the meteorite is big enough, it will pass through the atmosphere without burning up completely. When it strikes the surface of the Earth, its velocity and KE go to zero in an instant, but the law of conservation of energy holds that the energy is not simply lost; it is transferred to the surroundings as heat, light, and work, sending out shock waves and excavating a crater far larger than the meteorite itself.

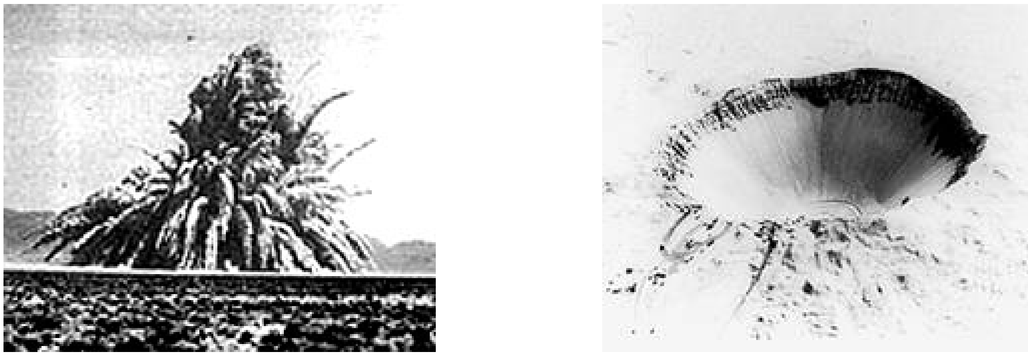
Consider the factors that would determine how "large" a crater is formed. In part, this would depend on geological conditions specific to the impact site, such as the mechanical properties of soils and rocks. However, one might also assume that KE of the meteorite is a more important factor: the more energy delivered upon impact, the "bigger" the crater that is excavated. (We will use *diameter* to represent crater "size," because as a crater is eroded away through time, its diameter changes far less than its depth.)

If KE is the most important (controlling) factor, and we can find a mathematical relationship between KE and crater diameter, we can take the dimensions of Meteor Crater and calculate the KE, and then the *mass*, of the offending meteorite. From the mass, we can then calculate the "size" (more precisely the *volume*) of the meteorite, because volume and mass are related by *density*, and we have actual fragments of the meteorite on which density has been measured.

Gather Data:

What data are available to help us solve this problem?

The impact that formed Meteor Crater is beyond history; evidence suggests that it occurred about 50,000 years ago. An impact of this size has not been observed on Earth in recorded time (and most would likely consider that a good thing!). But in the last century, for better or for worse, human beings have devised and experimented with a process of comparable destructive power: *nuclear explosions.* Until the advent of treaties restricting the practice, nations tested nuclear weapons by detonating them at or just beneath the surface.



Sedan, NV nuclear testing explosion and resulting crater. Image source http://rst.gsfc.nasa.gov/

In the United States, most nuclear weapons testing took place at the Nevada Test Site, in the Mojave and Great Basin Deserts of south-central Nevada, on the homelands of the Newe (Western Shoshone) people. The Newe had no say in the testing, and continue to work for restoration of these lands.

Nuclear explosions often excavated craters identical to those attributed to meteorite impacts. TheKE released in these blasts was known to the weapons designers, so here was a relationship between energy and crater size. This information was eventually made public, and planetary geoscientists made use of this relationship to estimate the KE needed to excavate impact craters of various sizes and ages, on Earth and other planets.

Some of these data, for what are thought to be actual meteorite impact craters are tabulated here. Crater diameter is reported in meters (m) and KE in joules (J).

Crater	**Diameter(m)**	**Kinetic Energy of Impact (J) x 10^{18}**
Brent	3800	2.461
Deep Bay	12000	15.85
Boltysh	23000	310
Clearwater Lake West	32000	1000
Manicouagan	70000	14500
Sudbury	140000	205000

Source: Roddy: Dence et al., 1977.

We can use mathematical regression on these data to **derive an equation** relating these two variables:

Notation

KE *= kinetic energy (in J) released by the impact of the meteorite*

D = *diameter (in meters) of the resulting crater*

This will enable us to calculate KE of impact for any Earth crater of known diameter, such as Meteor Crater. However, our ultimate target (so to speak!) is the *volume* of the meteorite, and for that we will need two more equations:

Information:

This equation models ***kinetic energy,*** $KE = 0.5Ms^2$

where *M* **=** ***mass of the meteorite in kilograms (kg)***

s* = *velocity of the meteorite in meters per second (meters/sec)

How fast do meteorites typically travel? **The average is about** *20,000 meters/sec***,** so let's use that value for s.

Information

This equation models ***density,*** $\rho = \frac{M}{V}$

ρ = ***density of the meteorite in kilograms per cubic meter (kg/meters3)***
V = ***volume of the meteorite in cubic meters (meters3)***

Solve the Problem:
Use the online applets or your calculators to answer the questions below.

Problem 1: Make points from the data provided in the table: the first coordinate should be the diameter of the crater and the second coordinate should be the kinetic energy. Write the second coordinate as shown in the table, just remember that the real value for KE must be multiplied by 10^{18}.

Problem 2: Use power regression to find the equation of best fit.

(Round constants to 3 decimal places.)

Information and Notation:

The equation as given by the power regression applet has the general form $y = ax^b$

In this problem

- $y \cdot 10^{18}$ represents KE, the kinetic energy in Joules (J), and
- x represents ***D***, the diameter of the crater in meters

So the form is $y \cdot 10^{18} = aD^b \cdot 10^{18}$. '$a$' and '$b$' are constants determined by the regression of the data. The coefficient 'a' should be in scientific notation. In this case,

- $a = c \cdot 10^{-12}$ (with ***c*** in the form x.xxx).

We can express this equation as: $KE = y \cdot 10^{18} = (aD^b)10^{18} = (c \cdot 10^{-12})D^b \cdot 10^{18} = cD^b \cdot 10^6$

We will use this equation for craters with diameters between 500 and 140,000 meters, 500 < D < 140,000.

Information: Meteor Crater is about 1,200 meters in diameter.

Problem 3: Now you are ready to determine the size of the meteorite that formed Meteor Crater.

a. Use the regression equation you developed to find the kinetic energy (KE) of impact. (You can do these calculations in the Math Pad in the Math Tool Chest or calculator.)

b. Use the kinetic-energy equation to find the mass of the meteorite. As noted above, use s = 20,000 meters/sec. Record your answer.

c. Use the density equation to find the volume of the meteorite. The iron-nickel fragments found at the site have a density of about 7,800 kg/meters^3. Record your answer.

d. Compare the size of the meteorite to the size of something else familiar to you.

Assume The meteorite was approximately spherical as it plunged to Earth.

Information You can find its radius using the equation for the volume of a sphere,

$$V = \frac{4}{3}\pi r^3$$

where ***V*** = *volume of the meteorite in cubic meters (meters^3)*

r = *radius of the meteorite in meters*

π *≈ 3.14159,* ***In Math Pad type 'pi' for π***

Follow-up Questions

5. Review the concepts from geology that were used in this study.
6. Review the math you used to answer the questions above.
7. Do you think the equations you used give accurate estimates of the size of the meteor?

8. How do scientists find equations like this?